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Procedia IUTAM 22 (2017) 51 – 58

Procedia IUTAM

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IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems

Attractive domain of nonlinear systems with time-delayed feedback control and time-delay effects

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Abstract

The dynamics of system with time delay resides in an infinite dimensional state space. It is difficult to determine the domain of attraction of the steady state response in the infinite dimensional state space. A definition of the domain of attraction in the physical space is introduced, which is intuitive, and furthermore, it is numerically computable under the hypothesis that the time delay is short. The effect of time delay on the domain of attraction of the popular Duffing system is considered. It is found that the time delay has significant influence on the structure of domain of attraction. The width of attractive domain can be enlarged with a small time delay, however, the domain of attraction shrinks when the time delay increases and approach to the stability boundary.

Peer-review under responsibility of organizing committee of the IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems

Keywords: Domain of attraction, Time delay, Duffing system, Periodic oscillation

1. Introduction

Local stability analysis can determine the evolution of dynamical system when it starts from the initial point which locates in the neighbourhood of one steady state. However, in many applications, it is required to determine the evolution of dynamical system when it starts from any given initial point in its phase space[1,2], especially for the system with multiple steady states, in this case, it is required to determine which steady state the system converges to when it starts from the initial point locates in different region in the phase space[3,4]. The set of initial points, from which the system will converge to one steady state, is called the domain of attraction of this steady state[5,4], which is usually characterized by its volume and boundary structure. The classical Lyapunov function method could be a candidate to estimate the volume of the domain of attraction with some theoretical significance, however, when the domain of attraction is complicated with fractal boundary or intertwined structure, this method is not applicable. From the viewpoint of application and efficiency, numerical methods are preferable and widely used in the calculation of the domain of attraction[6]. Studies indicate the domain of attraction of real systems can exhibit various dynamical phenomena. Fractal boundary resulted from external forcing was observed[7]. Under a general set of conditions, forced

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Peer-review under responsibility of organizing committee of the IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems
doi:10.1016/j.piutam.2017.08.008

double-well Duffing equation can have riddled basin of attraction with property that any neighborhood of a point in one basin contains points belonging to the basin of another attractor [8]. A rapid and dramatic erosion and stratification of the safe basin of attraction was observed in a single-degree-of-freedom system with small parameter changes [9], and the phenomenon of fractal basin erosion was illustrated in coupled non-linear systems [10]. Intertwined basins of attraction was found in studying the attraction of prey-predator system [11], and the definition of intertwined basins of attraction was introduced [5,12].

Since finite information transmission and processing speed, time delay is inevitable in active controlled system, resulted from controller, actuator and filter [13,14], such system is modeled by delay differential equation (DDE). The evolution of DDE depends on both present state and past state, and the initial value resides in an infinite dimensional space. Time-delay effect on local stability and bifurcation has been fully studied in literature, it is indicated even small time delay can significantly influence the local dynamics of DDE [15,16]. Time delay can frequently renders system unstable and leads to complex dynamics, such as periodic oscillation, quasi-periodic motion, coexisting motions, and even chaos [13,17]. On the other hand, under certain conditions, time delay can be beneficial to system [18,19]. Since the infinite dimensional state space, to study time-delay effect on global dynamics of DDE is still a challenging work [20,21]. Effect of time delay on the boundary of the basin of attraction in a single graded-response neuron with a delayed excitatory self-connection was studied [22], it is shown the boundary of the basin of attraction can be influenced by time delay even when the local dynamics is remain unchanged, and the time-delay effect was illustrated in the parameter space of a family of initial conditions. A class of delay differential equations with non-monotone bistable nonlinearities was considered [23], some subsets of basins of attraction of these equilibria for the DDE are characterized by combining the idea of relating the dynamics of a map to the dynamics of a DDE and invariance arguments for the solution semiflow. A generalized notion of basin's volume is theoretically described in finite-dimensional Euclidean space [21] through the proposed technique which projects the infinite dimensional initial state space to a finite-dimensional Euclidean space by expanding the initial function along with different orthogonal or nonorthogonal basis. In studying the time-delay effect on the safe basins of a controlled parametrically excited system [24], constant functions are taken as the initial value due to the fact that there is not any signal to be returned into systems before initial time t_0, it is concluded that delayed feedbacks can be applied to control the extent of the erosion of safe basins. Initial function space of the systems with time-delayed feedback control was discussed [25], by choosing the time of control switch as the initial time, it is argued that the initial function is actually the solution trajectory of the ordinary differential equation without time-delayed feedback control, such that the initial function space can be projected to the corresponding physical state space with finite dimension.

To the best of authors' knowledge, there is no general definition of the domain of attraction of delayed differential equation (DDE). In next section of present paper, a general definition of the domains of attraction in the physical state space is introduced, which is intuitive and numerically computable under the hypothesis that the time delay is short. Then in section 3, time-delay effect on the domain of attraction of the popular Duffing system is considered. Some conclusions are drawn in the last section.

2. Domain of Attraction of DDE

Mathematical model in controlled mechanical system falls into the form of delay differential equation (DDE)

$$\ddot{x} = f(t, x, \dot{x}, x(t-\tau), \dot{x}(t-\tau)) \tag{1}$$

where $x \in \mathrm{R}^n$ are state variables, a dot represents derivative with respective to t, τ is time delay, and function $f(t, x, \dot{x}, x(t-\tau), \dot{x}(t-\tau))$ is Lipschitz with respective to x, $\dot{x}$, $x(t-\tau)$ and $\dot{x}(t-\tau)$. The evolution of Eq. (1) is not only dependent on present state $(x(0), \dot{x}(0))$, but also dependent on its past state $(x(t), \dot{x}(t)) = (\psi(t), \dot{\psi}(t))$ when $t \in [-\tau, 0)$. Thus, the initial value of Eq. (1) is a function vector $(\psi(t), \dot{\psi}(t)) \in \mathrm{C}([-\tau, 0], \mathrm{R}^n \times \mathrm{R}^n)$ with $(\psi(0), \dot{\psi}(0)) = (x(0), \dot{x}(0))$, and $\mathrm{C}([-\tau, 0], \mathrm{R}^n \times \mathrm{R}^n)$ is an infinite dimension space composed of all continuous function vectors defined in $[-\tau, 0]$.

Since the initial value of Eq. (1) belongs to infinite dimension space $\mathrm{C}([-\tau, 0], \mathrm{R}^n \times \mathrm{R}^n)$, it is still difficulty to determine the domain of attraction of one steady state in that infinite dimension space. From practical viewpoint in engineering and controlling, it is required to properly define the definition of attractive domain of DDE in the physical state space ($(x, \dot{x})$-space), and this definition should be geometric intuitive and numerical computable.