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Revenue and efficiency in multi-unit uniform-price auctions ☆

Michal Bresky [1]

Charles University, Economic Institute, Politickych veznu 7, Prague 1, Czech Republic

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ABSTRACT

This paper analyzes efficiency in a uniform-price multi-unit auction with a positive reservation price. I demonstrate that the reservation price is an important policy tool that may increase efficiency (or welfare) in multi-unit uniform-price auctions. I show that the higher the reservation price is, the higher is the seller's revenue and the higher is the efficiency of a final allocation of units that could be attained in a multi-unit uniform-price auction. The reservation price increases the bidder's equilibrium strategy in a specific way that is inherent to the uniform-price auction. Thus the reservation price effect on efficiency is in contrast to other auction formats; e.g., the reservation price decreases efficiency in the Vickrey auction and single-unit auctions with symmetric bidders. Therefore the main result can be added to the list of results from mechanism design and auction theory that fail to extend the single-unit/single-dimensional context to the multi-unit/multi-dimensional one.

1. Introduction

An auction is an exchange mechanism with asymmetric information. It can be treated as a game in which the seller offers one or more units (of the same type) to the participants. The seller does not know the bidder's value of any particular unit, but he can set up an explicit set of institutional rules determining resource allocation and prices on the basis of bids from the auction participants. In Vickrey auctions (Vickrey, 1961) bidders reveal the true valuation of each unit and the final allocation is efficient. Krishna (2002) formulates conditions when the equilibrium in a multi-unit auction is efficient. However, the equilibrium strategies in a multi-unit uniform-price auction do not satisfy these conditions (see Morgan, 2001).

The effect of reservation prices on a multi-unit auction is difficult to assess in general (see Zhan, 2008). I show that in a private value multi-unit uniform-price auction, a positive reservation price increases both efficiency and revenue. Therefore it can be added to the list of results from mechanism design and auction theory that fail to extend the single-unit/single-dimensional context to the multi-unit/multi-dimensional one, e.g., Armstrong (1996), Perry and Reny (1999), and Levin (2004). For a benchmark of an auction game I follow the model with symmetric risk-neutral bidders who have independent-private values and where the payment is a function of bids alone as suggested by McAfee and McMillan (1987). The only additional assumptions are that the seller offers more than one unit for sale and the bidders demand two units that I call "initial" and "subsequent." This model has been analyzed in Engelbrecht-Wiggans and Kahn (1998). I focus on the difference between the bidder's true value and the submitted bid which is called "bid shading" or "demand reduction" in the literature. In a uniform-price auction with no reservation price, a different shading in strategies on initial and subsequent units

E-mail address: michal.bresky@cerge-ei.cz.

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is present and prevents the seller from reaching a Pareto-efficient distribution of units. I show that when the seller sets some specific (optimal) reservation price, the difference in shading on the initial and subsequent unit decreases, which can prevent some inefficient allocations of units and, moreover, the seller gains higher revenue.

For illustration, imagine two bidders in an auction with 2 units for sale with zero reservation price. Each bidder has two values and submits two bids. I denote the values of the first bidder v_1, v_2, the values of the second bidder as v'_1, v'_2 and the bids of the first bidder as b_1, b_2 and of the second bidder as b'_1, b'_2. If the values are such that $v'_1 > v'_2 > v_1 > v_2$, then it is efficient if the first bidder wins both units. But in many cases equilibrium strategic behavior forces the bidders to submit bids with the ordering $b'_1 > b_1 > b'_2 > b_2$, and the seller does not allocate the 2 units efficiently. When the seller increases the reservation price above v_1, the second bidder does not submit a bid above the reservation price, and the first bidder wins both units, which is an efficient outcome.[2] At the same time revenue typically increases. This reasoning is valid for the multi-unit uniform-price and to some extent for other multi-unit auctions if $b_1 > b'_2$ and $v_1 < v'_2$. On the other hand, setting the reservation price too high introduces inefficiency when the supply is greater than the number of submitted bids (e.g., $v'_1 > R > v'_2 > v_1 > v_2$). In summary, the seller faces a trade off between these two sources of inefficiency and the total effect is ambiguous. In this paper I show that the expected efficiency typically increases when the seller increases the reservation price above 0 in the uniform-price auction.

In addition, the results of this paper also contribute to the literature on efficient multi-unit auction design. Krishna (2002, Proposition 13.3) argues that equilibrium strategies cannot be efficient if a shading difference across units is present.[3] The seller who uses submitted bid ordering to allocate units cannot attain efficient allocations when bidders use different shading across initial and subsequent units. This paper, additionally, supports Krishna's proposition that any means that decrease shading differences can improve efficiency of the final allocation. The reservation price is an example of such a mean in a multi-unit uniform-price auction when the number of bidders is small.

The paper is organized as follows. At first I discuss the relationship of this paper to other studies. Then, in the next section, I develop a model of a uniform-price auction for n bidders, k units of supply, and a reservation price R. I also derive expressions for the expected efficiency measures. In the rest of the paper, I disentangle two sources of inefficiency, show the general conditions when the optimal reservation price that maximizes efficiency is positive, and demonstrate this main contribution of the paper with a simple example. Finally, I briefly discuss generalizations and conclude the paper.

2. Relation to the multi-unit auction literature

Even in the early studies (e.g. Back and Zender, 1993; Vickrey, 1961) the authors argued that standard features of single-unit auctions cannot be easily extended to multi-unit environments. To design an efficient multi-unit auction is a difficult task in general (see Zhan, 2008 and Klemperer, 2000 for surveys, and Dasgupta and Maskin, 2000; Perry and Reny, 2005a, 2005b for specific cases), and is not attainable in specific cases (Jehiel and Moldovanu, 2001a; Ausubel and Cramton, 2002; Krishna, 2002; Morgan, 2001).

There are many studies in the literature that analyze the so-called bid shading or demand reduction effect (see Ausubel and Cramton, 2002; Krishna, 2002; Menezes and Monteiro, 2005 and Zhan, 2008). Also, in the experimental literature (see Kagel and Levin, 2001; List and Lucking-Riley, 2002 and Engelmann and Grimm, 2009) the bid shading was detected especially in the open uniform-price auction format. The bid shading effect is also present in Engelbrecht-Wiggans and Kahn (1998), who derive essential features of equilibrium strategies that I use as a benchmark model.

In the model designed by Back and Zender (1993) a positive reservation price is an important policy tool that diminishes the bid shading effect; i.e., the higher the reservation price is, the higher the seller's revenue is. An important contribution of my paper is that increasing the reservation price above zero not only diminishes the bid shading and increases the seller's revenue but, in addition, also improves efficiency, which has not been pointed out in the literature yet. Similarly to Jehiel and Moldovanu (2001b) I split sources of inefficiency between the misallocation effect and supply restriction effect that I denote as *MLoss* and *ULoss*, respectively, in Section 4.

If the seller sets a positive reservation price, then bid shading diminishes because each bidder strictly increases his subsequent bid (see Fig. 1) for any given subsequent value above the reservation price. Therefore, the difference in shading across the initial and subsequent unit decreases (cf. the dark area and the same area with the light shaded area added).

Fig. 1 illustrates that the smaller the shaded area is, the smaller is the misallocation inefficiency that I call *MLoss*. Note that if the shaded area fully disappears, then the *MLoss* is zero and the necessary conditions for efficient multi-unit auction provided by Proposition 13.3 in Krishna (2002) are satisfied. In other words, one can consider the shaded area as a kind of measure closely related to misallocation inefficiency. This source of inefficiency is diminished by the reservation price. On the other hand, the positive reservation price increases the *ULoss* because the bidders bid below the reservation price for their low values. In short, an essential prerequisite for the effect studied in this paper is that each bidder bids fairly below his value for some but not all units (cf. Morgan, 2001, p. 815).

[2] Throughout the text, the words "above" and "below" mean strictly above and strictly below.

[3] Krishna (2002) uses the term "symmetry in strategies across objects" to mean "absence of difference in shading across units" in my terminology. But I use the term symmetric strategies to mean the "symmetry in strategies across bidders only" excluding "symmetry in strategies across objects" to simplify the terminology of this paper.