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Spiral orbits and oscillations in historical evolution of empires

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HIGHLIGHTS

- Cultural resources for social evolution considered.
- Three variable extension of Turchin's equation obtained.
- Intricate patterns of society's rise and fall observed.

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To the memory of Kijae Cheon (1930–2016)

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ABSTRACT

We introduce the concept of metaasabiya, the second non-material resource, to the asabiya theory of historical dynamics. We find that the resulting three variable dynamical system has peculiar features such as repelling or attracting axes and spiraling orbits in the phase space. Depending on the initial state, the system can go through series of oscillatory rises and falls, mimicking the geopolitical evolution of real-world polities. These distinctive features, absent in conventional Lotka–Volterra type biological systems, reveal the hidden richness inherent in the asabiya theory.

1. Introduction

The success of mathematical biology has been so impressive that we naturally expect the emergence of the mathematical theory of the evolution of human societies. The expectation has been met with the asabiya theory of Turchin [1,2], who has written down a set of dynamical equations that describes the temporal evolution of the size of a polity and the *asabiya*, its non-material resource, a concept adopted from the classic study of medieval North African history by Ibn Khaldun, representing the social cohesion that plays the key role in the growth and the decay of a polity.

The set of equations turns out to be capable of describing a rise and fall of a polity thanks to the presence of a fixed point of the system which acts as a repeller in the phase space. Turchin then considered several polities, each described by its size and asabiya, interacting through a geographically connected network of polities, in similar fashion to earlier attempts based on the "geopolitical" theories [3]. The evolution of polities has shown intricate patterns of oscillations that represent repeated rise and fall of polities, which could even be made to resemble the real historical data, with proper tuning of parameters [4]. We can, then, pose a question whether the historical cycle of a nation's rise and fall is necessarily the result of their mutual

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interactions. Is it that a polity can rise and fall only once, if it is left more or less undisturbed by other competing polity? That is unlikely. It rather seems reasonable to us, that there is an internal mechanism for the oscillating fortunes of polities, even in the absence of its interaction with others, and there should be a set of equations that, by itself alone, could capture the essence of dynamical oscillations found in human societies [5].

In this article, we present one candidate for such models with an introduction of the second non-material resource, which we call *metaasabiya* to the Turchin's equation. The concept of metaasabiya is a mathematical implementation of the historical philosophy described in the works of Oswald Spengler [6] and Arnold Toynbee [7], who had both stressed the cultural dimension of the sustenance of a polity in its later stage. They had observed that the polities, after the waning of their initial impulse for the creative evolution, can still resort to an institutionalized ideological system – morphism of *Kultur* into *Zivilisation* in Spengler's terminology, and the emergence of the *universal state* and the *universal religion* in Toynbee's terminology – for their second stage growth. Indeed, it is generally acknowledged that institutionalized religion with its derivatives in the form of scientific learning always accompany the great rise of nations in its later "imperial" stage. If we are to quantify the effect of the religion, it should naturally be considered as a quantity separate from asabiya, a quantity which can rise after the emergence of asabiya. This is exactly the reason for our choice of the neologism "metaasabiya" to represent this quantity.

Interestingly, the reexamination of Ibn Khaldun's text reveals that the embryonic form of the concept of metaasabiya, as we have outlined, can already be found in his writings. He had described the quick emergence of elaborate court culture in the Arab empire after its great conquest of Mesopotamia, with detailed analysis of its effects, both positive as well as negative, on the social cohesion of the conqueror's polity.

In the following sections, we detail our population dynamical model with the asabiya and metaasabiya (Section 2), pin down the skeleton of its dynamics in terms of the invariant surface, and analyze the orbits in the phase space to find spiral motion around the repeller (Section 3), study the nature of the spiral by linearized map around the repeller (Section 4), and look into the structure of basin of attraction (Section 5). We end the article with a discussion (Section 6) on the implication of our findings and possible future direction of the study of our model.

2. Population dynamics with asabiya and metaasabiya

We consider a *polity*, a socio-political entity made up of people, such as a tribe, a city-state, a nation, which we characterize by three time-dependent quantities the size $A(t)$, the asabiya $S(t)$, and the metaasabiya $R(t)$. We can identify the size of a polity either with its population or by the territorial area it controls, or alternatively, by its economic strength measured, for example, in terms of its gross domestic product (GDP). The asabiya and the metaasabiya are both non-material "spiritual" resources of a polity that bind people together and help the society to grow. The asabiya, as considered by Ibn Khaldun and by Turchin, represents the social cohesion of a polity that helps to increase its growth through higher birth rate, through the absorption of surrounding "barbarian" population, and through the military victories over rival polities. When the polity's asabiya is lost, often as a result of overabundance of elite population and loss of fiscal discipline, the growth rate of polity decreases and even become negative. Metaasabiya represents "cultural" assets of the polity, such as religion, science, art and entertainment, that work positively for the growth of a polity both by easing its internal tensions and providing technological advances. Metaasabiya, understood in this way, can also directly contribute to the increase of the polity's size by assimilating foreign elements in and around peacefully.

We consider the set of evolution equations given by

$$\begin{aligned} \frac{dA}{dt} &= \frac{c}{1+f}(S+fR)A\left(1-\frac{1}{h}A\right)-a, \\ \frac{dS}{dt} &= r\left(1-\frac{1}{2b}A\right)S(1-S), \\ \frac{dR}{dt} &= q\left(1-\frac{1}{2d}A\right)SR(1-R). \end{aligned} \tag{1}$$

In the first equation, the term $A(1-A/h)$ describes the logistic evolution of the polity size A whose growth limit is given by the quantity h determined by the natural resource of the land. The factor $S+fR$ signifies the necessity of the presence of either the asabiya S or the metaasabiya R as the condition for the growth, where the parameter f gives the ratio of respective contributions. The second term $-a$ gives the threshold, and when the first term is below a, population declines.

The parameters b and d, in the second and third equations respectively, are the key quantities of the system: The asabiya S, that quantify the internal cohesion of the polity takes the value $0 \leqslant S \leqslant 1$. The growth rate of asabiya is higher for smaller size of the population and it becomes zero at the critical population $A = 2b$. The metaasabiya R, a new concept of this article, also takes the value $0 \leqslant R \leqslant 1$. It quantifies the cultural resource of the polity. The law governing the growth of metaasabiya is similar to that of asabiya (with the critical population $2d$) except that the presence of larger value of asabiya is the prerequisite for the growth of metaasabiya. We can naturally assume that the saturation point of cultural assets, the metaasabiya $2d$ is higher than that for social cohesion, the asabiya $2b$. In this article, we mainly consider the case of $2b < \frac{h}{2}g < 2d$, which turns out to produce most interesting results. The parameters c, r and q gives the scale for the growth