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Tight focusing of radially polarized beams modulated by a fractal conical lens

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ABSTRACT

A novel high numerical aperture (NA) focusing system with a fractal conical lens (FCL) is proposed, and tight focusing of radially polarized beams through the proposed optical system is investigated theoretically and numerically. The influence of several relevant factors, including the FCL's stage S, objective lens' NA, and truncation parameter β_0, on the targeted beam's focusing characteristics in the focal region is discussed in detail. It is found that, when a FCL with $S \geq 0$ is employed, position of the major focal point would shift from the geometric focal point, and the focused intensity distributions cannot maintain symmetrical about the focus any more, although they present different profiles for various truncation parameters β_0. When $S \geq 2$, multiple focal points can be generated, i.e., a single major focus and a series of subsidiary foci surrounding it along the optical axis, which form a focal region. These unique focusing characteristics with a FCL are remarkably different from that of without a FCL. The fascinating findings here may be taken advantage of when using radially polarized beams in exploiting new-type optical tweezers and making use of a FCL.

1. Introduction

In recent years, tight focusing of radially and azimuthally polarized light beams through a high numerical aperture (NA) objective lens has attracted intensive attention and extensive investigation, because of their fascinating features and wide potential applications [1–10]. It is found that, when radial polarizations are highly focused, a very strong longitudinal electric field component will emerge in the focal region, which forms an extremely tight focal spot. It is known that a tight focal spot is of great help to improve the resolution of microscopy [11,12], enhance laser cutting ability in material processing [13], and can also be applied to improve the performance of optical tweezers due to the reduction of scattering [10,14,15]. Tight focusing of the electromagnetic field with various polarizations have been investigated, such as the linearly polarized, circularly polarized, radially polarized, azimuthally polarized, and hybridly polarized beams, as well as vortex beams [1–7]. It is also revealed in the vicinity of the focus that, for linear incident polarization, the generated longitudinal polarized component is not rotationally symmetric, which causes an asymmetric deformation of the focal spot [16,17]; for radial polarization input, it generates a strong longitudinal electric field component in the focal zone [11,18]; In contrast, the azimuthal incident polarization produces a strong magnetic field on the optical axis [19], meanwhile the electric field is purely transverse and null at the center [2]. When the hybrid polarized vector beams are highly focused, the focal shape may change from an elliptical spot to a ring focus with increasing the radial index, and meanwhile, the radial-variant spin angular momentum (SAM) of hybrid polarized vector beams is shown to be converted into radial-variant orbital angular momentum (OAM) [20]. One most recent report indicates that by dressing spatially variant polarization optical beams (e.g., azimuthally or radially polarized ones) with a vortex, one can generate at the focal plane subwavelength structures rotating with the optical frequency [21].

As a more general optical device than a conical lens (CL), the fractal conical lens (FCL) has its radial phase distribution following the Cantor function [22,23]. Since its first demonstration by Monsoriu in 2006, research and applications on a FCL has attracted considerable interest due to the particular self-similarity characteristics under monochromatic illumination [22–25]. In the present work, we propose a novel high numerical aperture (NA) focusing system, in which a fractal conical lens (FCL) is employed, and investigate the tight focusing of radially polarized beams through the proposed optical system. This paper is structured as follows. In Section 2, the theory model about the tight focusing of radially polarized beams modulated by a FCL is

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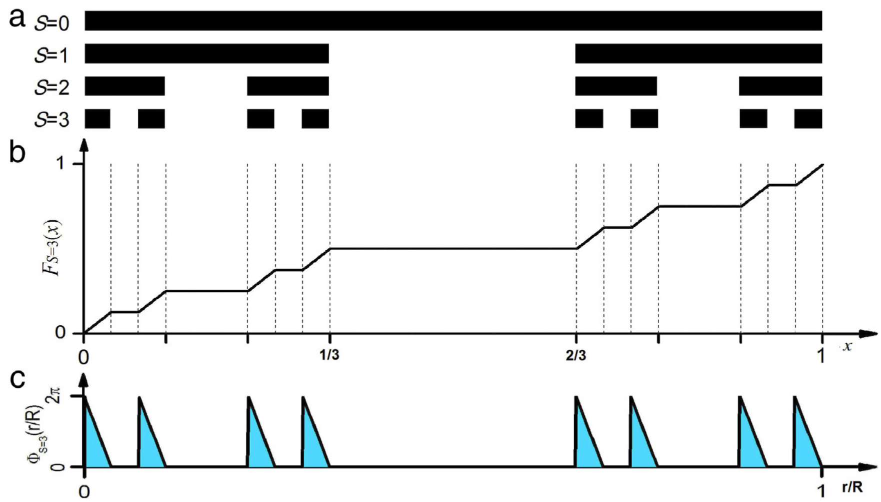

Fig. 1. (Color online) (a) Illustration of the generation of the triadic Cantor set, starting from the initiator, $S = 0$, to $S = 3$; (b) the Cantor function $F_S(x)$ for $S = 3$; (c) the phase of a triadic FCL for $\Phi_s = 2^{s+1}\pi F_s(\varsigma)$ with $S = 3$ and $\varsigma = r/R$.

presented, and the main focusing formula are derived. In Section 3, in terms of the formula obtained above, the influence of several relevant factors, including the FCL's stage S, objective lens' NA, and truncation parameter β_0, on the targeted beam's focusing characteristics in the focal region is numerically simulated and analyzed in detail. Finally, the main findings obtained are summarized in Section 4.

2. Theory model

As a new type of cylindrically symmetric diffractive lens, a fractal conical lens (FCL) has its radial phase profile designed from any given Cantor set (CS). As an example, Fig. 1(a) illustrates the construction of a regular triadic CS. The first step consists in defining a straight-line segment of unit length called the *initiator* (stage $S = 0$). Next, at stage $S = 1$, the generator of the set is constructed by dividing the segment into three equal parts of 1/3 and removing the central one. Then the procedure is continued at the subsequent stages $S = 2, 3, \ldots$. It is easy to find that, in general, at stage S there are 2^S segments of length 3^{-S} and $2^S - 1$ disjoint gaps located at intervals $[p_{S,l}, q_{S,l}]$, with $l = 1, \ldots, 2^S - 1$. For instance, $S = 3$, the triadic CS presents seven gaps at [1/27, 2/27], [3/27, 6/27], [7/27, 8/27], [9/27, 18/27], [19/27, 20/27], [21/27, 24/27], and [25/27, 26/7]. For clarity, the three first stages CS are depicted in Fig. 1(a). It should be noted that, a similar procedure could be followed for CS other than triadic.

Based on the fractal structure, the Cantor function $F_S(x)$ is defined in the domain [0, 1] as [22,23]

$$F_S(x) = \begin{cases} \dfrac{l}{2^S} & \text{if } p_{S,l} \le x \le q_{S,l} \\ \dfrac{1}{2^S}\dfrac{x - q_{S,l}}{p_{S,l+1} - q_{S,l}} + \dfrac{l}{2^S} & \text{if } q_{S,l} \le x \le p_{S,l+1}, \end{cases} \quad (1)$$

where S is the stage of the Cantor function, and l is the number of disjoint gaps intervals $[p_{S,l}, q_{S,l}]$ that the function has. Here q and p denote the start and end points for each segment of the Cantor function, respectively, and $F_S(0) = 0$ and $F_S(1) = 1$. For example, when $S = 3$, on the intervals, the constant values of $F_3(x)$ are 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, and 7/8, respectively (see Fig. 1(a) and (b)). In between these intervals the continuous function increases linearly, as plotted in Fig. 1(b).

The FCL is a rotational symmetric pupil whose phase profile is designed from the Cantor function of a given stage S. Then, the phase of a FCL of stage S is given by

$$q(\varsigma) = q_{FCL}(\varsigma, S) = \exp\left[-\mathrm{i}2^{S+1}\pi F_S(\varsigma)\right], \quad (2)$$

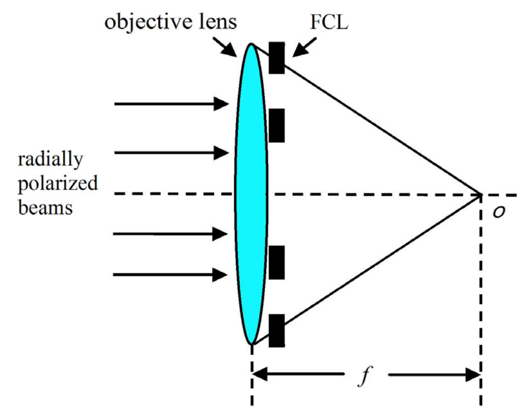

Fig. 2. Schematics of highly focused radially polarized beams modulated by a FCL.

where

$$\varsigma = r/R \quad (3)$$

is the radial variable r normalized by the pupil radius R. Then, the surface-relief profile of the FCL can be expressed from the relation [26]

$$h_{FCL}(r) = \bmod_{2\pi}\left[-2^{s+1}\pi F_S\left(\frac{r}{R}\right)\right]\frac{\lambda}{2\pi(n-1)}, \quad (4)$$

where $\bmod_{2\pi}[\phi(r)]$ denotes the phase function $\phi(r)$ modulo 2π, n represents the refractive index of the optical material used for constructing the lens, and λ is the wavelength of the light.

Fig. 1(c) illustrates the profile of a triadic FCL generated by Eq. (4) in the case $S = 3$. One can find that there exist eight phase peaks in the domain [0, 1] along the radial direction, and the phase profile changes linearly versus the normalized radial coordinate r/R.

When illuminated by a monochromatic light beam, the FCL's phase transmittance can be expressed as

$$T(\theta) = \exp\left[-\mathrm{i}2^{S+1}\pi F_S\left(\frac{\sin\theta}{\sin\alpha}\right)\right], \quad (5)$$

where θ is the convergence angle, and $\alpha = \sin^{-1}\left(\mathrm{NA}/n_1\right)$ denotes the maximum convergence angle determined by NA and n_1. Here NA is the objective lens' numerical aperture, and n_1 is the refractive index in image space.

In this work, we investigate the intensity distribution of highly focused radially polarized beams modulated by a FCL. In order to visualize the focusing procession, Fig. 2 illustrates the geometry of the