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Generalized regret based decision making

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A B S T R A C T

We describe the basic regret decision-making model for decision problems in which the payoff for a given alternative is uncertain and depends on the value of a variable called the state of nature. An important attribute of this model is the maximum payoff for the occurrence of a given state of nature. We note that the regret is based on the difference between the payoff we receive, given our choice of alternative, under the occurrence of a state of nature and the best payoff we could have received for that state of nature. We note the effective regret associated with an alternative is an aggregation of an alternative's regrets across all the possible states of nature. Our objective is to select the alternative with minimum effective regret. We look at this for various methods of aggregating an alternative's individual regrets across the different states of nature and various types of information about the uncertainty associated with the states of nature. One issue limiting the use of regret decision-making is its lack of indifference to irrelevant alternatives and the related openness to strategic manipulation by introducing alternatives to solely effect the determination of the maximum payoff. We begin to look at methods to reduce this effect.

1. Introduction

Regret decision theory has its roots in the work of the psychologists Kahneman and Tversky (1979, 1982). As opposed to classic decision making the fundamental imperative of regret decision-making is that of making decision choices with the goal of trying to minimize the dissatisfaction that occurs in not making the best decision. Early formal approaches to decision making under uncertainty that take into account anticipated regret were developed by Loomes and Sugden (1982), Bell (1982) and Fishburn (1982). Numerous other researchers have extended these early works (Zeelenberg et al., 1996; Gilovich et al., 1998; Yager, 2004a, b; Filiz-Ozbay and Ozbay, 2007; Bikhchandani and Segal, 2011; Bleichrodt and Wakker, 2015; Halpern and Leung, 2015).

The basic regret decision-making model is used for problems in which the payoff for a given alternative is uncertain depending the value of the state of nature. Central to the difficulty is the uncertainty with regard to information that would tell us what is the so-called state of nature. An important attribute of this model is the maximum payoff for the occurrence of a given state of nature. We note that the regret is based on the difference between the payoff we receive given our choice of alternative under the occurrence of a state of nature and the best payoff we could have received for that state of nature. We define the effective regret associated with an alternative as an aggregation of an alternative's regrets across all the possible states of nature. Our objective is to select the alternative with minimum effective regret. We look at this for various methods of aggregating an alternative's individual regrets across the different states of nature and various types of information about the uncertainty associated with the states of nature. One issue limiting the use of regret decision-making is its lack of indifference to irrelevant alternatives and the related openness to strategic manipulation by introducing alternatives to solely effect the determination of the maximum payoff. We begin to look at methods to reduce this effect.

We emphasize that this type of decision-making is particularly applicable in engineering problems in which we desire to build systems or agents to emulate human decision making, a notable example of this is robotics.

2. Regret type decision making

Consider an uncertain decision problem with a structure as shown in matrix shown below

$$DM = \begin{array}{cccc} & S_1 & S_j & S_q \\ A_1 & & & \\ A_i & & a_{ij} & \\ A_n & & & . \end{array}$$

In this structure A_i for $i = 1$ to n are a collection of alternatives one of which we must chosen, S_j for $j = 1$ to q are a set of possible states of

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nature and a_{ij} is the payoff to the decision maker if he selects alternative A_i and the state of nature is S_j. In this situation we are faced with a problem of decision-making in the face of uncertainty, as we do not know the state of nature before we must select our decision alternative.

Here we shall use as our decision imperative the desire to ***minimize the regret*** associated with our choice of decision alternative (Loomes and Sugden, 1982; Bell, 1982). Under this perspective we note that $V_j = \text{Max}_i[a_{ij}]$ is the maximal payoff that we can obtain if state nature S_j occurs. The classic definition of the regret associated with our selecting alternative A_i if the state of nature turns out to be S_j is $r_{ij} = V_j - a_{ij}$. This regret is the difference between the payoff we would have obtained, V_j, if before choosing an alternative we knew the state of nature S_j and we selected the best alternative under this state and the payoff we obtain selecting A_i under the occurrence of S_j.

In the following we show the typical regret matrix

$$\mathbf{RM} = \begin{array}{c} \\ A_1 \\ A_i \\ A_n \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_j & S_q \end{array} \\ \left[\begin{array}{ccc} & & \\ & r_{ij} & \\ & & \end{array}\right] \end{array}.$$

Illustration: Here we illustrate the calculation of the regret matrix, RM, associated with a decision matrix DM

$$DM = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \left[\begin{array}{ccccc} 20 & 40 & 35 & 10 & -40 \\ 15 & 10 & -20 & 20 & -10 \\ -10 & 20 & -30 & 30 & 25 \\ 30 & -50 & 15 & 40 & 0 \\ 15 & 0 & 25 & 50 & 20 \\ 20 & 20 & 20 & 20 & 20 \end{array}\right] \\ \begin{array}{ccccc} 30 & 40 & 35 & 50 & 25 \\ V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \end{array}$$

$$RM = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \left[\begin{array}{ccccc} 10 & 0 & 0 & 40 & 65 \\ 15 & 30 & 55 & 30 & 35 \\ 40 & 20 & 65 & 20 & 0 \\ 0 & 90 & 20 & 10 & 25 \\ 15 & 40 & 10 & 0 & 5 \\ 10 & 20 & 15 & 30 & 5 \end{array}\right] \end{array}.$$

Under decision-making using the paradigm of minimal regret for each alternative we calculate $R_i = \text{Agg}_{j=1 \text{ to } q}[r_{ij}]$ and then select the alternative A_{i^*} such that $R_{i^*} = \text{Min}_{i=1 \text{ to } n}[R_i]$.

The form of Agg depends on the information we have regarding the uncertainty associated with the states of nature and the imperative for combining the regret for the different states of nature. We shall refer to R_i as the effective regret under alternative A_i. Then our objective is to select the alternative with the minimal effective regret.

3. Ignorance about the state of nature

Here we shall consider the situation where we have no information regarding the state of nature. In this situation the formulation of R_i, the effective regret of an alternative, is solely dependent on the imperative used for combining an alternative's individual regrets for the different states of nature. One approach in this situation is to calculate

$$R_i = \underset{j=1 \text{ to } q}{\text{Agg}}[r_{ij}] = \underset{j}{\text{Max}}[r_{ij}].$$

Thus here the effective regret for an alternative is the maximal regret we can have under the choice of this alternative.

Here we are taking the worst possible case of regret. In this case our choice A_{i^*} is the alternative with the ***minimum maximal regret***,

$$R_{i^*} = \underset{i}{\text{Min}}[\underset{j}{\text{Max}}[r_{ij}]].$$

Another possible aggregation of an alternative's individual regrets under the different states of nature in this case of no information about the uncertainty associated with the different states of nature is

$$R_i = \underset{j=1 \text{ to } q}{\text{Agg}}[r_{ij}] = \left[\frac{1}{q}\sum_{j=1}^{q} r_{ij}\right].$$

Here we are taking R_i as the average regret associated with alternative A_i. In this case

$$R_{i^*} = \text{Min}_i\left[\frac{1}{q}\sum_{j=1}^{q} r_{ij}\right].$$

Here the selected alternative is the one with the least average regret.

More generally we can use the OWA operator (Yager, 1988; Yager et al., 2011) to provide a parameterized family of operations that can be used to aggregate the individual regrets associated an alternative. The OWA operator provides as easy way to implement properties we want associated with the aggregation of the individual regrets associated with an alternative.

Definition. Associated with an OWA operator of dimension q is a collection of q weights $w_k \in [0,1]$ for $k = 1$ to q such that $\sum_{k=1}^{q} w_k = 1$. If for $j = 1$ to q the y_j are a collection of numeric values then the OWA aggregation of these values is

$$OWA(y_1, \ldots, y_q) = \sum_{k=1}^{q} w_k y_{\rho(k)}$$

where ρ is an index function so that $\rho(k)$ is the index of the kth largest of y_j. Thus $y_{\rho(k)}$ is the kth largest of y_j.

We see that the OWA operator provides a weighted average of its arguments, it is essentially a mean of the argument values (Beliakov et al., 2007). We shall refer to the vector $W = [w_1, \ldots, w_q]$ as the OWA weighting vector. The type of weighted average, mean, is determined by the choice of weighting vector.

In the framework of regret type decision-making we can use

$$R_i = OWA(r_{i1}, r_{i2}, \ldots, r_{ij}, \ldots, r_{iq}) = \sum_{k=1}^{q} w_k r_{i\rho_i(k)}.$$

In this format $\rho_i(k)$ is the index of the kth largest regret under alternative A_i. Thus $r_{i\rho_i(k)}$ is the kth largest regret for alternative A_i. We emphasize that the index function ρ_i is specifically defined for each alternative.

We note here that if $w_1 = 1$ and $w_k = 0$ for all $k \neq 1$ then $R_i = \text{Max}_j[r_{ij}]$ and if $w_k = 1/q$ for all k then $R_i = \frac{1}{q}\sum_j r_{ij}$. The OWA operator can provide many different formulas for the determination of R_i. For example if $w_q = 1$ and all $w_k = 0$ for $k \neq q$ then $R_i = \text{Min}_j[r_{ij}]$. Here the effective regret associated with an alternative is the minimum regret. In the framework of decision making using regret this does not appear as a reasonable choice for r_i. Thus we see that not all OWA aggregations, i.e., weighting vectors W, are appropriate for use in calculation of the effective regret. As we noted the $R_i = \sum_{k=1}^{q} w_k r_{i\rho_i(k)}$ is effectively a weighted average of regrets for alternative A_i. Here we see that $r_{i\rho_i(k)}$ is the kth largest regret for alternative A_i. One condition we want in the aggregation is that we assign no more weight to a smaller regret then to the bigger regret. This implies that we have $w_{k_1} \geq w_{k_2}$ for $k_1 \leq k_2$.

We can easily show that the weighting vector with $w_1 = 1$ and $w_k = 0$ for $k \neq 1$ provides the largest value of R_i for any OWA weights satisfying $w_{k_1} \geq w_{k_2}$ for $k_1 \leq k_2$. Similarly $w_k = 1/q$ for all k provides the smallest value for R_i for any set of OWA weights satisfying $w_{k_1} \geq w_{k_2}$ for $k_1 \leq k_2$.

Example 1. Let us look at R_i for our illustrative regret matrix for these two notable cases of OWA vector.

(a) $w_1 = 1$ and $w_k = 0$ for $k \neq 1$. Here $R_i = \underset{j=1 \text{ to } 5}{\text{Max}}[r_{ij}]$. In this case

$$R_1 = 65, R_2 = 55, R_3 = 65, R_4 = 90, R_5 = 50, R_6 = 30.$$

Using this aggregation imperative A_6 is the alternative with the minimal regret.