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Regret theory: State dominance and expected utility

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HIGHLIGHTS

- We present a holistic method of comparing expected utility (*EU*) and regret theory (*RT*).
- The *SD* method allows comparison of *RT* and *EU* when risk aversion is assumed.
- We identify important cases where dominance under *EU* and *RT* coincides.
- We identify cases where dominance patterns under *CPT* and *RT* coincide.
- Our study has implications for the design of experimental tests of *RT*.

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ABSTRACT

Choices made according to regret theory (*RT*) may violate the expected utility (*EU*) model. We propose a stochastic dominance (*SD*) method for comparing *RT* and *EU* paradigms holistically, without focusing on a specific axiom or on a specific numerical example. We show that in some important cases, including the two-state case, e.g., war or peace, Republican or Democratic Party winning the Presidential election etc., *RT* does not violate both *EU* and Cumulative Prospect Theory (*CPT*). Obviously, when *EU* is not violated by *RT* the economic results derived under *EU* are intact also under *RT*.

1. Introduction

Expected utility (*EU*) theory, cumulative prospect theory (*CPT*) and regret theory (*RT*), are the three main competing decision-making models under conditions of uncertainty. *EU* theory, being a normative theory, prescribes the choices one *should* make, given certain axioms. In contrast, although the *RT* and *CPT* theories have some normative ingredients, they are basically descriptive theories of how people make their choices, based on the choices typically made by subjects in laboratory experiments. There is no agreement in the literature about the paradigm that best fits choices as observed in experimental studies and as revealed by empirical economic phenomena. It has been argued that the results of laboratory experiments in some specific cases violate *EU* theory. It has also been claimed that observed experimental choices fit *RT* better than *CPT* (Loomes, Starmer, & Sugden, 1991, 1992; Loomes & Sugden, 1982).

Generally, evaluating two prospects with uncertain monetary payoffs by *RT* and by *EU* may yield a different ranking. Specifically, the choice by *RT* may contradict *EU*, which in turn, contradicts classic economic theory. Bleichrodt and Wakker (2015) who thoroughly analyze and compare *RT* and *EU*, provide some evidence supporting *RT*, as well as some evidence contradicting *RT*. Thus, relying mainly on the subjects' choices with hypothetical numerical examples, we find that: (1) in some cases there is a support for *RT*, (2) in some cases there is a support for *CPT*, and (3) in some cases the observed choices support *EU* theory. Hence, there is no one theory without drawbacks. Despite the above mentioned disagreement, some researchers argue that feelings like regret and rejoice, which are the pillars of *RT*, are a fact of life, therefore it is irrational to ignore them (see Bleichrodt & Wakker, 2015). We adopt this approach in this study.

The purpose of this study is to identify conditions under which *RT* and *EU* agree, and alternatively conditions under which *RT* and *CPT* agree. We employ stochastic dominance (*SD*) rules, which are

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commonly employed in economics, to identify cases where *RT* and *EU* (and alternatively *RT* and *CPT*) yield the identical ranking of prospects. The employed stochastic dominance rules are general and do not rely on a specific utility function or on specific numerical examples. Furthermore, to achieve the results reported in this paper there is no need to know the precise shape of the regret function. We prove that if some conditions imposed on the distributions of the outcomes of the two prospects under consideration are fulfilled, then *RT* and *EU* provide the same prospect ranking. In the main part of this paper the only assumptions made are that the regret function and the utility functions are increasing. If the ranking by *RT* and *EU* coincides (and there are many important cases where they do), then, there is no contradiction between the feelings of regret and rejoice, as captured by *RT*, and between maximizing *EU*, as classic economic theory advocates.

Finally, recall that if prospect *A* does not dominate prospect *B* by first degree stochastic dominance (*FSD*), this implies that for some utility functions *A* is preferred and for other utility functions *B* is preferred, hence in this case any choice by *RT* does not contradict the *EU* paradigm. Therefore, we focus here on the case where *A* *does* dominate *B* by *FSD*, yet also by *RT* (for all increasing regret functions) prospect *A* is optimal; hence there is no contradiction between *RT* and *EU* theory. However, recall that if *A* dominates *B* by *FSD* it implies that *A* also dominates *B* by *CPT* (see Levy, 2015); hence in this case if *RT* and *FSD* agree on prospect ranking, then *RT* and *CPT* also agree.

In this paper we do not endorse any particular interpretation of the various experimental results (in particular we take no position on whether they violate some specific *EU* axiom); instead, our view is that each of the three theories has pros and cons. Indeed, from the numerous examples given by Bleichrodt and Wakker (2015) it emerges that there is no one decision making model that is optimal for all scenarios. Comparing two prospects, *A* and *B*, we analyze cases where prospect ranking by *RT* and *EU* paradigms, and alternatively, by *RT* and *CPT* paradigms coincides. We then extend the analysis by comparing *RT* to *EU* ranking when risk aversion is assumed.

Our main finding is that there are important scenarios where the optimal *EU* and *RT* choices coincide. However, we would like to emphasize at the outset that we are unable to disentangle *RT* and *EU*, but can only identify non-trivial cases where the two approaches yield an identical prospect ranking. For example, some subjects may make their choices by *RT* and some subjects by *EU*, but this is economically irrelevant so long as the two groups of subjects make the same choice. Thus, the goal of this paper is not to find the best theory, but rather to identify cases where no conflict arises between the various pairs of theories.

The remainder of the paper is organized as follows. Section 2 provides a brief literature review. Section 3 presents the competing decision-making theories and some definitions of concepts and decision rules employed in this study. Section 4 explains *RT* and introduces the concepts of state dominance violation (*SV*) and state dominance violation correction (*SVC*). Section 5 compares and analyzes the required dominance conditions under *EU* and *RT* frameworks, and alternatively in *CPT* and *RT* frameworks. Section 6 discusses the theoretical and methodological implications of the theoretical results. Section 7 presents some conclusions. The mathematical proofs are given in the appendices.

2. Literature review

The expected utility (*EU*) theorem (von Neumann & Morgenstern, 1953) is fundamental to most economic models; it sets out four axioms that must be satisfied if a decision-maker's choice is to be optimal i.e. maximizes *EU*: completeness, independence, continuity and transitivity (Fishburn, 1982). If one of the axioms is violated, then *EU* will not necessarily be maximized. Early studies highlighted some instances in which choices made by subjects in experimental studies are inconsistent with the *EU* model (Allais paradox, Allais, 1953; Ellsberg paradox, Ellsberg, 1961), presumably because one (or more than one) of the axioms was violated. Numerous experimental studies have demonstrated that preferences are intransitive. The transitivity axiom, which must hold if choices are to maximize *EU*, is that given three prospects *x*, *y* and *z*. if $x \succ y$ and $y \succ z$ then $x \succ z$ must hold. The transitivity of preferences is a fundamental assumption underlying most theoretical and empirical economic research; hence its violation has a direct effect on many economic results which assume transitivity.

The validity of the transitivity axiom has been investigated in numerous experimental studies. Some early studies suggested that preferences were intransitive (Tversky, 1969) and some more recent studies have shown that when subjects are required to make a sequence of choices their choices do not always obey the transitivity axiom. There is evidence that violations of transitive preference are not always due to random error and it has been argued that such violations are consistent with *RT* (Loomes et al., 1991). There is, however, another way of explaining choices - including intransitive choices - that does not rely on *RT*. It is called the 'heuristic priority rule' (Brandstätter, Gigerenzer, & Hertwig, 2006).

RT and *CPT* assume that subjects making decisions under condition of uncertainty do not always act as efficient maximizing machines; in other words emotions and other factors, which economists consider irrelevant to financial choices do, at least sometimes, affect choices. There are numerous studies showing how the so-called irrational factors influence choices. Furthermore, some of these studies also analyze the economic implications of the observed choices. (Barberis, Huang, & Thaler, 2006; Filiz-Ozbay & Ozbay, 2007; Gollier & Salanié, 2006; Muermann, Mitchell, & Volkman, 2006; Perakis & Roels, 2008).[1]

Not all researchers agree that the results cited do represent violations of the transitivity axiom. It has been claimed that the results of many of the studies designed to elicit intransitive preferences are in fact consistent with transitive preference (e.g. Regenwetter, Dana, & Davis-Stober, 2011). Others, who suggest a novel approach for testing transitivity, find evidence of transitive preferences and hence argue that the transitivity axiom has not been violated (Baillon, Bleichrodt, & Cillo, 2014 p. 199). Specifically, there are studies in which the subjects' choices deviate from *EU* theory, but the authors argue that this is not because the subjects displayed intransitive preferences and conclude that transitivity should not be abandoned.

Thus, despite apparently convincing experimental evidence for *RT*, some authors have cast doubt on the interpretation of the experimental results cited in support of *RT*, arguing that they are due to other factors such as framing, event splitting, etc. rather than to the regret–rejoice effect (Battalio, Kagel, & Jiranyakul, 1990; Birnbaum, 2008; Harless, 1992; Starmer & Sugden, 1993). It is worth noting that choices may fail to maximize *EU* even if the transitivity axiom is not violated. For example, prospect theory (*PT*) experiments have revealed, amongst other things, that subjects make choices based on *change* of wealth rather than *total* wealth (which contradicts *EU*) and that they employ decision weights $w(p)$ rather than probabilities p (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). Thus, although *CPT*, a modified version

[1] Some of the models of observed choices simply modify the classic *EU* model by adding other economic variables in addition to personal wealth, e.g., suggesting that preference is a function of relative wealth (see Campbell & Cochrane, 1999), or replacing the univariate utility function with a bivariate function, where one variable is the individual's wealth and the other is his or her peer group's wealth. Numerous studies have explored this bivariate framework, which is widely known as the 'keeping up with the Joneses' model (cf. Abel, 1990).