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Optimal recursive estimation for networked descriptor systems with packet dropouts, multiplicative noises and correlated noises

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ABSTRACT

This paper addresses the optimal linear estimation problem for a class of networked descriptor systems with multiple packet dropouts, measurement multiplicative noises and finite-step correlated process and measurement noises. Based on a fast–slow subsystem decomposition approach (FSD), the descriptor system is transformed into two reduced-order linear nonsingular subsystems with finite-step correlated noises. Optimal linear estimators including filter, predictor and smoother with corresponding estimation error covariance matrices for the states and noises of new systems are developed via the innovation analysis approach. Then, the optimal linear estimators are obtained for the original descriptor system. An example shows the effectiveness of the proposed algorithms.

1. Introduction

Descriptor systems or singular systems often appear in robotics, economics, electrical circuit networks, power systems, aerospace engineering, biomedical sciences, chemical industry and many other practical application fields. They have more extensive description forms than non-singular systems. Therefore, the estimation and control problems of descriptor systems have been paid great attention to in the past few decades [1–6].

There are a great number of approaches to deal with the state estimation problems of descriptor systems. For example, the reduced-order Kalman filters have been investigated in [7–13] based on singular value decomposition (SVD) and fast–slow subsystem decomposition (FSD) [1,5]. On the basis of the modern time-series analysis method, the time-domain Wiener filters for descriptor systems are presented in a unified framework [13]. In [14], the full-order state estimator for lower dimension descriptor systems has been studied. In [15], the original descriptor system is transformed into the equivalent non-singular systems with unknown inputs by using full-row rank decomposition and the optimal state estimator is achieved. When the noise statistics information of descriptor systems is unknown, the self-tuning reduced-order and full-order filters are given by identifying the parameters of an ARMA innovation model [16]. In [17], a robust Kalman filter is proposed for descriptor systems with uncertainties. By taking the data quantization into account, the event-triggered controller of networked singular system is designed in [18].

Traditional Kalman filtering algorithm assumes that both the process noise and measurement noise are uncorrelated or correlated white noises at the same moment. In practical applications, finite-step correlated noises are often caused due to the continuous system discretization [19], the model transformation of the networked systems with random delays [20], the same noise source of environmental pollution and other reasons in the systems. The estimation problem with finite-step correlated noises is always complex and challenging. The Kalman-type filters with finite-step correlated process and/or measurement noises have been discussed in [19,21]. However, they are suboptimal because the filters are fixed as Kalman-like forms. In [22,23], the distributed Kalman fusion estimators are studied for a class of uncertain systems with one-step auto-correlated and/or two-step cross-correlated noises. Meanwhile, the networked control systems (NCSs) have also received much attention due to the advantages of low cost, great mobility, simple installation and implementation. In NCSs, random time delays and packet dropouts or missing measurements often appear in data transmission [24–31]. Moreover, networked systems with multiple delays and packet dropouts can be transformed into those with finite-step correlated noises [20] or random parameterized systems [25–31]. Nowadays, it has been a hot research area to take into account networked uncertainties and correlated noises simultaniously [32–39]. The optimal and suboptimal filters for networked systems with multiple packet dropouts and correlated noises are designed in [32–35]. However, the above literatures are mainly focused on non-descriptor systems. The corre-

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sponding results for descriptor systems are relatively few. In recent study [36], Feng et al. have presented the recursive estimation for descriptor systems with multiple packet dropouts and correlated noises. The centralized and distributed fusion state estimators are also designed for the corresponding multi-sensor systems with multiplicative noises, correlated additive noises and networked uncertainties in [37–39]. However, one-step auto- and/or two cross-correlated noises are only concerned in [36–40], but multi-step auto- and cross-correlated noises are not taken into account. Moreover, main results in most literature focus on filter design, but multi-step predictor and smoother are seldom reported.

Motivated by the above discussion, to the best of the authors' knowledge, the optimal estimation problem has not been solved for the descriptor systems with multiple packet dropouts, multiplicative noises and finite-step correlated noises. It has wide applications in practice such as circuits, networks and chemical industry. By using FSD and innovation analysis approach [41], we will derive the optimal linear estimators including filter, predictor and smoother with the estimation error covariance matrices for the state and noises of descriptor systems. The proposed estimation algorithms generalize the existing results in some present literature. Compared with the existing results, the main contributions of this paper are emphasized as follows: 1) both finite-step auto- and cross-correlated noises are taken into account in descriptor systems; 2) optimal linear estimators including filter, predictor and smoother with corresponding estimation error covariance matrices are presented for descriptor systems; 3) the system model comprehensively considers some issues of multiple packet dropouts, multiplicative noises and correlated noises.

The rest of the paper is organized as follows. In Section 2, the problems studied are stated. In Section 3, the networked descriptor system is transformed into two reduced-order nonsingular systems and the statistical properties of the systems are analyzed. In Section 4, the optimal linear state and noise estimators are developed. In Section 5, an example is presented to illustrate the effectiveness of the algorithms. Some conclusions are drawn in Section 6 and Appendices provide the mathematical details.

Notations. Throughout this paper, $\boldsymbol{R}^n$ and $\boldsymbol{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all real $n\times m$ matrices, respectively. $\boldsymbol{C}$ is the set of complex numbers. $\mathrm{E}(x)$ is the expectation of random variable x. P^{T} is the transpose of matrix P. $\det(A)$ is the determinant of matrix A. $\mathrm{diag}\{\cdot\}$ denotes the block diagonal matrix. $\perp$ denotes orthogonality. $\mathrm{Prob}\{*\}$ denotes the occurring probability of event $*$. I_n and 0 represent the $n\times n$ identity matrix and zero matrix with appropriate dimensions, respectively. $\det(A)$ is the determinant of matrix A. $\delta_{t,k}$ is the Kronecker delta function. In addition, $L(y(t), y(t-1), \cdots, y(0))$ stands for the linear space spanned by $(y(t), y(t-1), \cdots, y(0))$. $\hat{x}(j|t)$ means the linear minimum variance estimate of the vector $x(j)$ based on the $L(y(t), y(t-1), \cdots, y(0))$. $\tilde{x}(j|t)$ is the estimation error, i.e., $\tilde{x}(j|t) = x(j) - \hat{x}(j|t)$.

2. Problem formulation

Consider the following networked descriptor control system with multiple packet dropouts and measurement multiplicative noises

$$\begin{aligned} &Mx(t+1) = \Phi x(t) + Bu_e(t) + \Gamma w(t) \\ &z(t) = \left(H + \sum_{i=1}^{n_a} \zeta_i(t)\bar{H}_i\right)x(t) + v(t) \qquad (1) \\ &y(t) = \xi(t)z(t) + \big(1-\xi(t)\big)y(t-1) \end{aligned}$$

where t is the discrete time, $x(t) \in \boldsymbol{R}^n$ is the state, $u_e(t) \in \boldsymbol{R}^u$ is the known control input, $z(t) \in \boldsymbol{R}^m$ is the measurement output to be transmitted to the estimator, and $y(t) \in \boldsymbol{R}^m$ is the measurement received by the estimator. The multiplicative noise $\zeta_i(t) \in \boldsymbol{R}$, $i = 1, \ldots, n_a$ are the scalar white noises that describe the uncertainties of measurement matrix parameters. Bernoulli distributed stochastic variable $\xi(t)$ is uncorrelated with other noise signals, with the probabilities $\mathrm{Prob}\{\xi(t) = 1\} = \alpha$ and $\mathrm{Prob}\{\xi(t) = 0\} = 1-\alpha$. The third equation in (1) can describe the phenomena of multiple packet dropouts in networked systems. Moreover, this model has been widely used in recent many references [31–33]. Clearly, it follows that $y(t) = z(t)$ if $\xi(t) = 1$ (i.e., received on time); on the contrary, $y(t) = y(t-1)$ if $\xi(t) = 0$ (i.e., the measured data $z(t)$ is lost and the latest measurement $y(t-1)$ received at time instant $t-1$ is used as a compensator). So, α is called receiving rate on time and $1-\alpha$ packet loss rate (PLR). Φ, B, Γ, H and $\bar{H}_i, i = 1, \ldots, n_a$ are known constant matrices with appropriate dimensions.

Assumption 1. M is a singular square matrix, i.e., $\det(M) = 0$.

Assumption 2. The system is regular. i.e., for $\forall z_0 \in \boldsymbol{C}$, $\det(z_0 M - \Phi) \neq 0$.

Assumption 3. $w(t) \in \boldsymbol{R}^p$ and $v(t) \in \boldsymbol{R}^m$ are β_0-step ($\beta_0 \geq 1$) correlated noises with zero mean and

$$\begin{aligned} &\mathrm{E}\big[w(t)w^{\mathrm{T}}(k)\big] = Q_w(t,k), \\ &\mathrm{E}\big[w(t)v^{\mathrm{T}}(k)\big] = \bar{S}(t,k), \\ &\mathrm{E}\big[v(t)v^{\mathrm{T}}(k)\big] = Q_v(t,k) \end{aligned}$$

where $Q_w(t,t) = Q_w(t), \bar{S}(t,t) = \bar{S}(t)$ and $Q_v(t,t) = Q_v(t)$. For $k > t+\beta_0$ or $k < t-\beta_0$, we set

$$Q_w(t,k) = 0, \qquad \bar{S}(t,k) = 0, \qquad Q_v(t,k) = 0.$$

Assumption 4. The multiplicative noise $\zeta_i(t) \in \boldsymbol{R}$ is the scalar white noise with zero mean and is uncorrelated with other noise signals. $\zeta_i(t)$ has the following statistical properties:

$$\mathrm{E}\big[\zeta_i(t)\zeta_j^{\mathrm{T}}(k)\big] = Q_{\zeta_i}(t)\delta_{t,k}\delta_{i,j}$$

Our aim is to find the linear minimum variance recursive state estimators $\hat{x}\ (t|t+N)$, where for $N = 0$, $N > 0$ or $N < 0$, it is called descriptor state filter, smoother or predictor, respectively.

Remark 1. From the distribution of $\xi(t)$, it can be easily obtained that

$$\begin{aligned} &\mathrm{E}\big[\xi(t)\big] = \alpha, \qquad \mathrm{E}\big[\xi^2(t)\big] = \alpha, \qquad \mathrm{E}\big[\big(\xi(t)-\alpha\big)^2\big] = \sigma^2, \\ &\mathrm{E}\big[\xi(t)\big(\xi(t)-\alpha\big)\big] = \sigma^2, \qquad \mathrm{E}\big[\xi(k)\xi(t)\big] = \alpha^2, \quad t \neq k, \end{aligned}$$

where the variance $\sigma^2 = \alpha(1-\alpha)$.

Let $J(t) = \xi(t) - \alpha$, then the standardized stochastic variable $J(t)$ is a white noise with zero mean and variance σ^2.

3. System transformation and statistical property analysis

3.1. System transformation

Using Assumption 2, there exist non-singular matrices P and Q [1, 5] such that

$$PMQ = \mathrm{diag}\{I_r, \Omega\}, \qquad P\Phi Q = \mathrm{diag}\{\Phi_1, I_{n-r}\},$$

where $\Omega \in \boldsymbol{R}^{(n-r)\times(n-r)}$ is a nilpotent matrix with the nilpotent index γ ($\gamma \geq 1$), and then we define