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# Achieving arbitrarily polygonal thermal harvesting devices with homogeneous parameters through linear mapping function

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ABSTRACT

Transformation optics based metamaterial thermal harvesting devices, which provides power and efficiency related advantages by manipulating and concentrating thermal fluxes, have attracted considerable research interests. The potential utilizations are significant for improving the energy conversion efficiencies in existing collecting technologies including heat storages, solar thermal collectors, full cells, etc. However, most of the current researches on thermal harvesting devices are mainly focused on continuous shape profiles based on the scattering cancellation. In this paper, we propose a novel method for fabricating desirable polygonal thermal harvesting devices with homogeneous and nonsingular parameters through linear mapping function. Four polygonal concentrating schemes with fewer kinds of natural materials filling inside the functional regions are demonstrated in transient states. The expected harvesting performances are achieved for each proposed polygonal scheme. Furthermore, the proposed thermal harvesting performances are investigated as a function of the dense degree of the medium-layer configurations. In general, this paper demonstrates a novel method for designing non-continuous shape polygon harvesting devices with corresponding validations. Improved concentrating behaviors would be achieved with denser medium configurations, implemented as appropriately arranged fractions. The findings could have potentials for designing novel transformation optics devices and providing improved energy conversion efficiencies in existing collecting technologies.

## 1. Introduction

Owing to the advances in transformation optics [1] (TO) and conformal mapping [2], significant achievements including the development of novel invisible cloaks, highly efficient concentrators, camouflages, lenses, etc., have been promoted in various physical fields based on the form invariance of the governing functions after coordinate transformations. Using pre-designed coordinate transformations, the flow of electromagnetic waves can be precisely manipulated by mapping tailored properties onto relevant spatial transformations. Upon such configurations of material properties, invisible cloaks have been theoretically and experimentally demonstrated in the fields of electromagnetics [1,3] and optics [4,5]. Regarding the electric and magnetism effects on the manipulation processes, DC and magnetic cloaks [6] and concentrators [7] have been proposed to regulate the currents and fluxes. Owing to the expansions of the applications of transformation optics, mechanical metamaterials [8] and novel acoustic devices [9] have also been motivated. As the form invariance of the heat conduction equation has been verified [10], the cloaking performances were also exhibiting in the thermal field through engineering thermal materials [11] and establishing relevant shaped thermal cloaks [12–15] upon efficient medium theory. These studies have promoted the developments of artificial thermal metamaterial, in which the conductivities distribute non-uniformly.

Over the last few years, the techniques of energy harvesting, which can be employed to collect ambient energies, such as the heat [16], light [17], fuel [18], thermoelectricity [19], and vibration [20], have attracted a considerable research interest. Benefiting from the TO-based thermal cloak, a similar theory, named transformation thermodynamic [21], has been proposed to guide the general regulations of heat flux, and the concept of the thermal concentrator (regarded as the reverse cloak [22]) has also been demonstrated. Such new class of thermal concentrators is believed to occupy a key place in significantly improving thermal conversion efficiencies of conventional functions, which represents its unique capability of efficiently harvesting and focusing thermal energy without severe thermal perturbations outside the devices [15]. In order to achieve efficient concentrating behavior, which is independent of the geometrical size, a theoretical and general

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## Nomenclature

| | |
|---|---|
| $a$ | expansive/compressive component of $x$ in $x'$ direction |
| $b$ | expansive/compressive component of $y$ in $x'$ direction |
| $c$ | expansive/compressive component of $z$ in $x'$ direction |
| $d$ | expansive/compressive component of $x$ in $y'$ direction |
| $e$ | expansive/compressive component of $y$ in $y'$ direction |
| $f$ | expansive/compressive component of $z$ in $y'$ direction |
| $J$ | Jacobian matrix |
| $N$ | side number of polygon |
| n | serial number of each element in same function type |
| $\mathbf{R}(\theta)$ | rotation matrix of step 1 |
| $r_0$ | inside radius in transformation domain after step 2 (m) |
| $r_1$ | inside radius in original domain after step 1 (m) |
| $r_2$ | outside radius in both original and transformation domain (m) |
| $T$ | temperature (K) |
| $x$ | $x$ direction in the original domain (m) |
| $y$ | $y$ direction in the original domain (m) |
| $x'$ | $x$ direction in the transformational domain after step 1 (m) |
| $y'$ | $y$ direction in the transformational domain after step 1 (m) |
| $x''$ | $x$ direction in the transformational domain after step 1 (m) |
| $y''$ | $y$ direction in the transformational domain after step 1 (m) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | composited fractions of the related material layers (%) |
| $\beta$ | total fractions of positive or negative mediums |
| $\theta$ | rotation between original and principle axis system (°) |
| $\kappa$ | thermal conductivities in the original domain ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |
| $\kappa'$ | thermal conductivities after step 1 ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |
| $\kappa''$ | thermal conductivities after step 2 ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |
| $\kappa_\mathrm{A}$ | thermal conductivity of material layer A ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |
| $\kappa_\mathrm{B}$ | thermal conductivity of material layer B ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |
| $\kappa_\mathrm{b}$ | thermal conductivities of background medium ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |
| $\kappa_\mathrm{N}$ | thermal conductivity of negative medium ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |
| $\kappa_\mathrm{P}$ | thermal conductivity of positive medium ($\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) |

*Subscripts*

| | |
|---|---|
| I | function elements I |
| II | function elements II |
| sc | proposed polygonal schemes |
| pl | contrast bare plate |

design methodology based on scattering-cancellation [23] was proposed for fabricating thermal concentrators using natural materials without singularities. Moreover, a class of wedge thermal metamaterial units [24,25] made of natural materials were demonstrated, which were capable of controlling thermal flux in different ways through using certain quantities. More recently, bifunctional TO devices including camouflages [26] and concentrators [27,28] were developed, in order to simultaneously manipulate the Laplace fields (DC and thermal fields). In addition, the entropy generation rates [29–31] considering such coordinate transformations were proposed to estimate relevant performances in transient states. However, the profiles of the above proposed devices were shaped in common, i.e. following continuous TO functions (circle, cylinder or sphere). Scattering-cancellation cannot be employed to design irregular or arbitrary shaped profiles [32], as well-established effective medium theory for regular geometries [23–31] play the predominant role during the design process. For achieving a non-continuous shape profile TO device and avoiding material singularities, quasi-conformal mapping was employed to achieve a novel class of thermal cloak, called ground (carpet) cloak [32], designed by regulating the anomalous refraction bends [33,34] in metamaterials. Although non-continuous shape profile can be obtained using quasi-conformal mapping, two aspects restrict its applications, i.e., the quasi conformal profiles of the devices and functional regions, and the narrow-warped regions for transformation processes. Furthermore, the linear mapping function was used to map the material characteristics between the linear segments and fabricate N-beam emissions [35], and homogeneous illusion [36] in electromagnetic field. According to such methodology, the arbitrarily polygonal thermal cloaks [15] considering inside rotated region, the novel thermal lens [37] for remote cooling or heating, and the 3D thermal illusion for location camouflage [38] were theoretically proposed. However, the complicated processes of determining appropriate characteristic lines and the employments of local coordinate systems are challenging for the practical designs of TO concentrators, though the material singularities can be avoided. Considering the potential applications of thermal harvesting devices, the appropriate design method that simultaneously avoids singularities and quasi conformal profiles, and optimizes the transformation process of the linear mapping function, is of particular importance in fabricating highly efficient arbitrary shape concentrators.

In this paper, an advanced methodology for designing arbitrarily polygonal thermal harvesting devices is proposed in order to develop polygonal thermal harvesting schemes with nonsingular and homogenous material parameters. In addition, space rotations are employed in the transformational designs to determine the characteristic lines, which allows the entire process to be operated in one global system. A detailed analysis of employed appropriate materials with certain composited fractions is performed to select fewer kinds of natural materials filled in the functional regions. Four different polygonal schemes, i.e., square, pentagon, hexagon, and heptagon schemes, are demonstrated in transient states in order to validate the proposed derivations. Furthermore, the thermal harvesting behaviors in such polygonal schemes are investigated as a function of the wedge material layer numbers inside the one functional region, by comparing the temperature gradients inside the concentrating regions and temperature deviations of the heat-fronts.

## 2. Theoretical design and obtainment of geometrical models

### 2.1. Theoretical derivation of conductivities tensor based on transformation optics

On the basis of the general design of TO devices [1], the combinations of the original and transformational domains are realized through Jacobi matrix. The transformation process can be written as:

$$\kappa' = \frac{J\kappa J'}{\det(J)}. \tag{1}$$

where $\kappa$ and $\kappa'$ denote the thermal conductivities in the original and transformational domains, respectively. $J$ is the related Jacobi matrix used to combine the transformation process, which can be expressed as follow:

$$J = \frac{\partial(x',y',z')}{\partial(x,y,z)} = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} & \frac{\partial x'}{\partial z} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial z} \\ \frac{\partial z'}{\partial x} & \frac{\partial z'}{\partial y} & \frac{\partial z'}{\partial z} \end{pmatrix} = \begin{pmatrix} a_i & b_i & 0 \\ d_i & e_i & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

In Eq. (2), $x$, $y$, and $z$ represent the direction parameters in the original domain, $x'$, $y'$, and $z'$ denote the corresponding values in the transformation domain. For simplicity and generality, we employ a 2D