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Theoretical determination of gamma spectrometry systems efficiency based on probability functions. Application to self-attenuation correction factors

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ABSTRACT

A generic theoretical methodology for the calculation of the efficiency of gamma spectrometry systems is introduced in this work. The procedure is valid for any type of source and detector and can be applied to determine the full energy peak and the total efficiency of any source-detector system. The methodology is based on the idea of underlying probability of detection, which describes the physical model for the detection of the gamma radiation at the particular studied situation. This probability depends explicitly on the direction of the gamma radiation, allowing the use of this dependence the development of more realistic and complex models than the traditional models based on the point source integration. The probability function that has to be employed in practice must reproduce the relevant characteristics of the detection process occurring at the particular studied situation. Once the probability is defined, the efficiency calculations can be performed in general by using numerical methods. Monte Carlo integration procedure is especially useful to perform the calculations when complex probability functions are used. The methodology can be used for the direct determination of the efficiency and also for the calculation of corrections that require this determination of the efficiency, as it is the case of coincidence summing, geometric or self-attenuation corrections. In particular, we have applied the procedure to obtain some of the classical self-attenuation correction factors usually employed to correct for the sample attenuation of cylindrical geometry sources. The methodology clarifies the theoretical basis and approximations associated to each factor, by making explicit the probability which is generally hidden and implicit to each model. It has been shown that most of these self-attenuation correction factors can be derived by using a common underlying probability, having this probability a growing level of complexity as it reproduces more precisely the geometric and attenuation configuration of the source-detector system. Experimental verification of the improvement produced by increasing the model complexity has been performed by measuring samples spiked with certified nuclide activities with a HPGe detector. The methodology can be extended to obtain the theoretical efficiencies and corrections corresponding to any geometry by defining adequately the physical model and the subsequent probability corresponding to the particular studied situation.

1. Introduction

Mathematical methods are an important tool to determine the efficiency of gamma spectrometry systems. The theoretically determined efficiencies can be used in practice both directly for activity calculations and also indirectly to determine different corrections factors. Different direct analytical methods have been developed to calculate the efficiency [1–3], and also Monte Carlo codes are nowadays available to calculate the efficiency for a wide variety of sources and detectors arrangements [4,5]. However, there is no common conceptual frame to describe these methods using a single theoretical formalism. In general, most of these methods start from the assumption of specific conditions and approximations that are different from one to another analytical method. It is therefore sometimes very difficult to compare these different theoretical or numerical methodologies and to identify the origin of the encountered differences in calculations [6–8].

The aim of this job is to present a generic methodology that could help to derive the different theoretical methods using a single formalism, based on the probability of detection, as follows. In order

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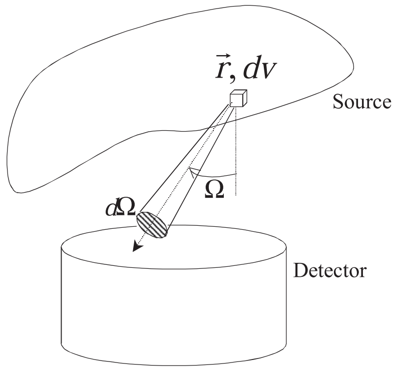

Fig. 1. Gamma radiation emitted by a generic shape volume source.

to derive the integral expression of the efficiency which is the central issue of this work, lets consider the gamma radiation emitted by a generic shape source with volume V (Fig. 1). Lets define $p(\vec{r}, \Omega)$ as the probability for a photon emitted from the position $\vec{r}$ and having an angular initial direction specified by Ω to be detected. This probability is also a function of the gamma energy, that is $p = p(E, \vec{r}, \Omega)$, but this dependence will not be written explicitly in order to simplify notation, and because we are interested in highlight the geometric dependence of the probability. It must be emphasized that Ω is not a solid angle, being actually a simplified notation to express the emission direction of the photon, which could be explicitly written in a spherical coordinate system through the polar and azimuthal angles θ, φ respectively. However, dΩ will be used to denote the differential solid angle (having angular direction Ω) in which photons are emitted. This solid angle could also be explicitly written in spherical coordinates as dΩ = sinθ dθ dφ.

Defining $dN(\vec{r}, \Omega)$ as the number of photons generated in a differential volume dv and having initial direction inside the solid angle $d\Omega$, the number of detected (counted) photons $dD(\vec{r}, \Omega)$ associated will be

$$dD(\vec{r}, \Omega) = p(\vec{r}, \Omega)\ dN(\vec{r}, \Omega) \tag{1}$$

This expression can be understood as the definition of the probability itself, $p(\vec{r}, \Omega)$ being the fraction of detected photons coming from the position $\vec{r}$ and initial direction Ω, and showing therefore that this probability really exists, even though in practice usually it is not explicitly known.

As dN is a fraction of the total number of photons N emitted by the sample, assuming an homogeneous distribution of the radionuclide in the sample and isotropic emission from $\vec{r}$, we have

$$dN(\vec{r}, \Omega) = N\frac{dv}{V}\frac{d\Omega}{4\pi} \tag{2}$$

and so the detections $dD(\vec{r}, \Omega)$ can be written as

$$dD(\vec{r}, \Omega) = \frac{N}{4\pi V}p(\vec{r}, \Omega)\ dv\ \ d\Omega \tag{3}$$

The integration of this expression for the whole sample volume V and for every emission direction Ω is the total number of detections (counts) D. Therefore, considering the definition of the efficiency, $\varepsilon = D/N$, we have

$$\varepsilon = \frac{1}{4\pi V}\int_V \int_{4\pi} p(\vec{r}, \Omega)\ dv\ \ d\Omega \tag{4}$$

This expression is the central issue of the present work. This integral form of the efficiency is exact and valid for any type of source and detector. The only two hypothesis required for its validity are the homogeneous distribution of radionuclides in the sample and the isotropic emission of the gamma radiation, which are conditions generally fulfilled in conventional gamma spectrometric measurements. The dependence of the efficiency on the energy is implicitly contained in the probability, that is, actually $p = p(E, \vec{r}, \Omega)$ being therefore $\varepsilon = \varepsilon(E)$.

In order to determine ε using (4) a probability function $p(\vec{r}, \Omega)$ has to be proposed at the particular studied situation and then the integral has to be solved. This particular probability employed will always be an approximation, because due to the complexity of the detection process of gamma radiation (in particular, due to the second or higher order gamma interactions that finally deposit the energy in the detector) in general no exact explicit expression for $p(\vec{r}, \Omega)$ can be developed. The simplified probability that has to be proposed in practice should reproduce the most relevant facts of the detection process taking place at the particular studied situation. Anyway, it must be emphasized that even though the efficiency obtained this way is an approximation, the expression (4) itself, before using the particular approximation for the probability, is an exact and universal way of expressing the efficiency. Moreover, (4) is valid to determine the full energy peak efficiency (FEPE) and also for total absorption efficiency, since no particular hypothesis about the detection has been made to deduce the expression. Again, in practice, the determination of ε (full energy or total efficiency) has to be performed by defining properly the probability at the particular studied situation.

The idea of the present work is to show the utility and importance of the expression (4), showing that it can be used in practice to derive all the theoretical approximations for the efficiency that appears in literature, by proposing an appropriate simplification for the probability $p(\vec{r}, \Omega)$ at each particular situation. Using this single and universal formalism, the whole set of theoretical efficiencies and correction factors commonly used in gamma spectrometry can be obtained, ordering therefore the extraordinary variety of proposals that can be encountered in literature. The methodology is open, and can be used by any researcher to propose new “personalized” efficiencies and correction factors, just by defining adequately the probability $p(\vec{r}, \Omega)$ at the particular studied situation and solving the integral. As an example, some of the most important classical factors employed to calculate self-attenuation corrections for cylindrical geometry sources will be derived later.

2. Relationship between angular and classical point-source integral expressions

It is possible to deduce from (4) another integral expression for the efficiency, the so-called *point-source* integral expression for the efficiency, which is the classical expression usually employed in gamma spectrometry literature for the calculation of the efficiency via elemental integration. From (3), we can obtain the detections $dD(\vec{r})$ coming from the volume dv (at the position $\vec{r}$) regardless of the initial emission direction, by integrating (only) for every emission direction Ω

$$dD(\vec{r}) = N\frac{dv}{V}\frac{1}{4\pi}\int_{4\pi} p(\vec{r}, \Omega)\ d\Omega \tag{5}$$

On the other hand, following a similar reasoning as for $p(\vec{r}, \Omega)$, defining $\varepsilon_p(\vec{r})$ as the probability for the photons emitted from the position $\vec{r}$ to be detected (regardless of the emission direction), the detections $dD(\vec{r})$ can be written as

$$dD(\vec{r}) = \varepsilon_p(\vec{r})\ dN(\vec{r}) = \varepsilon_p(\vec{r})\ N\frac{dv}{V} \tag{6}$$

where the homogeneity of the source has been employed. This expression can be understood as the definition of the probabaility $\varepsilon_p(\vec{r})$, in a similar way as previously done for $p(\vec{r}, \Omega)$. Moreover, equaling expressions (5) and (6), it is obtained the relationship between these two probabilities

$$\varepsilon_p(\vec{r}) = \frac{1}{4\pi}\int_{4\pi} p(\vec{r}, \Omega)\ d\Omega \tag{7}$$

Expressing the double integral given by the expression (4) as