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Automatic variance ratio test under conditional heteroskedasticity

Jae H. Kim *

Department of Econometrics and Business Statistics, Monash University, Caulfield East, Vic. 3145, Australia

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ABSTRACT

An extensive Monte Carlo experiment is conducted to evaluate small sample properties of the automatic variance ratio test under conditional heteroskedasticity. It is found that the test shows serious size distortion in small samples. For improved small sample performance, this paper proposes the use of wild bootstrap. When wild bootstrapped, the automatic variance ratio test shows no size distortion, and it has power substantially higher than its competitors such as the Chen–Deo test and wild bootstrap Chow–Denning test.

1. Introduction

The variance ratio test has been used widely as a means of testing for the weak-form efficiency of financial markets, and of evaluating predictability of financial return. Recent empirical studies include Shamsuddin and Kim (2008) and Belaire-Franch and Opong (2005) on financial market efficiency; and Patro and Wu (2004) on financial return predictability. Since Lo and MacKinlay (1988) proposed its original form, the test has undergone a number of improvements. Recent contributions include the multiple variance ratio test of Chow and Denning (1993), sign and rank tests of Wright (2000), subsampling test of Whang and Kim (2003), wild bootstrap tests of Kim (2006), and power-transformed

* Fax: +61 3 99032007.
E-mail address: Jae.Kim@BusEco.monash.edu.au

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test of Chen and Deo (2006). A survey of recent developments of the variance ratio tests is given by Charles and Darne (2009).

The test is based on the property that, if an asset return is purely random, the variance of k-period return is k times the variance of the one-period return. Hence, the variance ratio $VR(k)$, defined as the ratio of $1/k$ times the variance of k-period return to that of one-period return, should be equal to one for all k. To implement the test, a choice of (holding periods) k values should be made; e.g., a popular choice for daily return is (2,5,10,20,40); and (2,4,8,16,32) for weekly returns. However, these choices are arbitrary and made with little statistical justifications. In response to this, Choi (1999) proposed an automatic variance ratio (*AVR*) test, in which the optimal value of k is determined using a completely data-dependent procedure. However, Choi (1999) reported small sample properties of the *AVR* test when the return follows an *iid* process, while its properties under conditional heteroskedasticity are completely unknown. The purpose of this paper is to evaluate the size and power properties of the *AVR* test when the return is conditionally heteroskedastic.

It is found that the *AVR* test should be wild bootstrapped for correct size in small samples. The power of the wild bootstrap *AVR* test is found to be higher than its competitors in small samples, such as the wild bootstrap version of the Chow–Denning test (Kim, 2006) and the power-transformed joint test of Chen and Deo (2006). The next section presents the *AVR* test and its wild bootstrap version. Section 3 presents alternative variance ratio tests, and Section 4 reports the Monte Carlo results.

2. Automatic variance ratio test under conditional heteroskedasticity

Let Y_t be an asset return at time t ($t = 1, \ldots, T$). Choi's (1999) *AVR* test is based on the statistic of the form

$$VR(k) = 1 + 2\sum_{i=1}^{T-1} m(i/k)\hat{\rho}(i) \tag{1}$$

where $\hat{\rho}(i) = \frac{\sum_{t=1}^{T-i}(Y_t - \hat{\mu})(Y_{t+i} - \hat{\mu})}{\sum_{t=1}^{T}(Y_t - \hat{\mu})^2}$ and $\hat{\mu} = T^{-1}\sum_{t=1}^{T} Y_t$, while $m(x) = \frac{25}{12\pi^2 x^2}\left[\frac{\sin(6\pi x/5)}{6\pi x/5} - \cos(6\pi x/5)\right]$ is the quadratic spectral kernel. Choi (1999) stated that $VR(k)$ in (1) is a consistent estimator for $2\pi f_Y(0)$, where $f_Y(0)$ is the normalized spectral density for Y_t at the frequency zero.

Choi (1999) showed that, under $H_0^A : Y_t$ is serially uncorrelated (or $H_0^B : 2\pi f_Y(0) = 1$),[1]

$$AVR(k) = \sqrt{T/k}[VR(k) - 1]/\sqrt{2} \xrightarrow{d} N(0,1) \tag{2}$$

as $k \to \infty$, $T \to \infty$, $T/k \to \infty$, when Y_t is *iid* with a finite fourth moment. He further stated (without proof) that the result in (2) holds when Y_t is generated from a martingale difference sequence with proper moment conditions. In order to choose the value of lag truncation point (or holding period) k optimally, Choi (1999) adopted a data-dependent method of Andrews (1991) for spectral density at the zero frequency. The *AVR* test statistic with the optimally chosen lag truncation point is denoted as $AVR(\hat{k})$.

The $AVR(\hat{k})$ test is an asymptotic test which may show deficient small sample properties. When Y_t is subject to conditional heteroskedasticity, the wild bootstrap of Mammen (1993) can be employed to improve small sample properties, as in Kim (2006) who applied the wild bootstrap to the Lo–MacKinlay and Chow–Denning tests. The wild bootstrap for $AVR(\hat{k})$ can be conducted in three stages as below:

(i) Form a bootstrap sample of T observations $Y_t^* = \eta_t Y_t$ ($t = 1, \ldots, T$) where η_t is a random sequence with $E(\eta_t) = 0$ and $E(\eta_t^2) = 1$;
(ii) Calculate $AVR^*(\hat{k}^*)$, the *AVR* statistic obtained from $\{Y_t^*\}_{t=1}^{T}$; and

[1] Note that H_0^A implies H_0^B, but the converse is not necessarily true (see, for details, Choi, 1999).