Journal of Computational and Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Territorial design optimization for business sales plan

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ARTICLE INFO

Article history:
Received 1 July 2017
Received in revised form 9 January 2018

Keywords:
Territory design
Projection methods
Optimization
Scheduling
Routing

ABSTRACT

A well designed territory enhances customer coverage, increases sales, fosters fair performance and rewards systems and lower travel cost. This paper considers a real life case study to design the sales territory for a business sales plan. The business plan consist of assigning the optimal quantity of sellers to a territory including the scheduling and routing plans for each seller. The problem is formulated as a combination of assignment, scheduling and routing optimization models. The solution approach considers a meta-heuristic using stochastic iterative projection method for large systems. Several real life instances of different sizes were tested with stochastic data to represent raise/fall in the customers demand as well as the appearance/loss of customers.

1. Introduction

Goals such as improving customer satisfaction ratings and achieving desired revenue levels come under the heading of management effectiveness. To help the decision-making process, managers use mathematical models to analyze the relevant data. Current state-of-art provides classical approaches to solve well known problems. Typically, in a real case, the cost efficient management decision is defined by a combination of different models.

This research considers a real life territory design problem. The main goals are to assign an optimal number of sellers to serve a set of customers located in a certain region, a definition of the weekly schedule plan, and the design of the daily optimal route for each seller. The decision should consider location and demand of the customers as well as daily capacity of the sellers to deliver goods. The mathematical model for the problem combines objectives and constraints of three classical approaches, the clustering of customers, the scheduling of visits, and the routing plan.

At its origin, Territorial Design is the process of allocating the population of a region to a set of service providers. Goodrum [1] provides a literature review of two algorithms developed and applied to case study areas. Both algorithms consider a fixed population organized as blocks and fixed located providers. The first algorithm provides an Agglomerative Optimization of facility selection. This algorithm selects a provider closest to the high population block. The second algorithm allows planners to view potential catchment areas for facilities based on facility capacity rather than equal population or distance criteria.

Consider the location of the customers (such as points in the area or nodes of a network) with a given distance between every pair of points. We wish to find a cluster of customers using the nearest neighbor approach. Therefore, each cluster will represent a seller in the solution. This objective can be interpreted as the tightest cluster of m points.

This problem is found in literature as one facility location problem. For a discrete formulation, the basic model is presented by Drezner [2], where one facility is to be located minimizing the maximum value of a function of demand using Euclidean

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https://doi.org/10.1016/j.cam.2018.02.010
0377-0427/

distance. Another model is the max-cover problem. The discrete formulation [3] of this problem considers fixing the number of facilities that are to be located and maximizing the number of covered demands. In a planar models [4], demand occur anywhere on a plane. Typically, demand is forecasted using a probability distribution, then facilities are located according to the mobility of the demand. For several facilities Watson [5] presents a model that aims to cover the maximum number of points within a given distance. In this model each point has an associated weight w, a critical distance R and an integer number m, The model determines the location of m centers so that the sum of the weights of those points lying within distance R of at least one center is maximized.

The following step in the procedure is the scheduling of the visits to customers. Effective scheduling systems aim at matching demand with capacity so that resources are better utilized and waiting times are minimized. Tuga and Emre [6] provide a comprehensive survey on appointment scheduling in outpatient services. The underlying problem applies to a wide variety of environments of outpatient scheduling, and is modeled using queuing system representing the unique set of conditions for the design of the patient appointment. The authors present a complete survey of problem definitions and formulations considering the nature of Decision-Making and Modeling of Clinic Environments. In addition, they mention a variety of performance criteria used in the literature to evaluate appointment systems, which are grouped as: (a) Cost-Based Measures, (b) Time-Based Measures, (c) Congestion Measures, (d) Fairness Measures.

Finally, the goal of the problem described above is to optimize the distribution process from depots to customers (routing design) in such a way that customer's demand of goods is satisfied without violating any problem-specific constraint. In the literature, these kind of logistic problems are known as Vehicle Routing Problems (VRP) and the objective regularly is the minimization of the complete distance traveled by the vehicles while servicing all the customers. The VRP is an interesting problem in operations research due to its practical relevance and the difficulty to be solved exactly. Moreover, it is one of the most demanding NP-hard problems [7]. In reality, the task of finding the best set of vehicle tours by solving optimization models has a high computational cost and is prohibitive for medium and large real life applications.

Caceres et al. [8] present a survey on VRP's applied to real life problems. The authors call these VRP's as Rich (realistic) VRP's (RVRP's) and classify their variants according to the company's decision levels and the routing elements involved. A classification that applies for this case study is Multi-Period/Periodic VRP with Multiple Visits/Split deliveries. In this classification, the clients are visited several times as vehicles may deliver a fraction of the customer's demand. Moreover, optimization is made over a set of days, considering a different frequency of visits to each client.

In this work, a real business strategy for sales in different territories is modeled using the formulation of three classical problem: cluster, the scheduling and VRP. Particularities of the modeling approach include scheduling constraints of visits spread over the week, service and traveling times, as well as time capacity to ensure the fulfillment of the clients demand.

The paper is organized as follows. A general mixed integer linear programming (MILP) formulation for the problem is presented in Section 2. Section 3 describes a projection method for large systems of linear equations and two optimization models that we use to solve the resulting smaller problems and thus enabling to reduce the solution time. In Section 4, the models are tested for different scenarios. Finally, conclusions are presented in Section 5.

2. Mathematical formulation

Consider a set of customers $C = \{1, 2, \dots, N\}$ denoted by indexes i and j dispersed in a given region with geographical coordinates $(long, lat)$. It is desired to design a business plan that assigns the customers to a set of sellers $S = \{1, 2, \dots, S\}$ denoted by index s. The sellers will satisfy the demand of customers during the weekdays $W = \{1, 2, 3, 4, 5, 6\}$ denoted by index t in the scheduling plan per week. Finally, it is desired to get the optimal daily routing. The distance between two location is computed using the Haversine formula given in Eq. (1).

$$d_{i,j} = 2R \arcsin\left(\sqrt{\sin^2\left(\frac{lat_j - lat_i}{2}\right) + \cos(lat_i)\cos(lat_j)\sin^2\left(\frac{long_j - long_i}{2}\right)}\right) \tag{1}$$

where latitude (lat) and longitude $(long)$ are given in radians, and R is Earth's radius (mean radius = 6371 km).

The Haversine formula determines the great-circle distance between two points on a sphere given their longitudes and latitudes. Important in navigation, it is a special case of a more general formula in spherical trigonometry, the law of Haversines, that relates the sides and angles of spherical triangles [9].

The mathematical formulation of the model is defined in the following equations:

The objective function, given in Eq. (2), represents the sum of two objectives, the minimization of the distance between customers assign to the sellers $d_{s,i}$ as well as the traveling distance to visit each customer for each routing plan $d_{i,j}$. This equation has two binary decision variables: y_i^s with value of 1 if the customer i is assigned to seller s, and $x_{i,j}^{s,t}$ with value of 1 if customer j is visited after customer i by seller s on day t.

$$\min\left[\sum_{s\in S}\sum_{i\in \mathbb{N}} d_{s,i} y_i^s + \sum_i\sum_j\sum_s\sum_t d_{i,j} x_{i,j}^{s,t}\right]. \tag{2}$$

As for constraints, Eq. (3) ensures that a customer is attended by only one seller.

$$\sum_{s\in S} y_i^s = 1; \qquad \forall i. \tag{3}$$