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On finite exchangeable sequences and their dependence

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Abstract

This paper deals with finite sequences of exchangeable 0–1 random variables. Our main purpose is to exhibit the dependence structure between such indicators. Working with Kendall's representation by mixture, we prove that a convex order of higher degree on the mixing variable implies a supermodular order of same degree on the indicators, and conversely. The convex order condition is then discussed for three standard distributions (binomial, hypergeometric and Stirling) in which the parameter is randomized. Distributional properties of exchangeable indicators are also revisited using an underlying Schur-constant property. Finally, two applications in insurance and credit risk illustrate some of the results.

Keywords: Actuarial risk models, convex order, exchangeable indicators, factorial series distribution, higher degree, supermodular order.

1. Introduction

A sequence of random variables is exchangeable when their joint distributions are invariant to permutations of the indices of the variables. Exchangeability corresponds to a form of dependence in which all the variables play a symmetric role. This notion introduced by de Finetti [7] plays an important role in probability and statistics.

When the number of exchangeable random variables is countably infinite, a key theorem of de Finetti states that their joint distributions are mixtures of iid distributions. This characterization, however, is no longer true when there are only finitely many random variables. We refer to Diaconis [13] and Diaconis and Freedman [14] for an estimation of the distance between an exchangeable distribution and mixtures of iid distributions.

The present work is concerned with finite sequences of exchangeable 0–1 random variables. A number of properties on such sequences can be found, e.g., in the book by Chow and Teicher [4]. Our main purpose is to analyze the dependence structure between the indicators. This will be accomplished using the concepts of convex and supermodular orders of higher degree. Basic distributional properties of exchangeable indicators are also revisited.

More precisely, the paper is organized as follows. Section 2 deals with Kendall's representation by mixture of finite sequences of exchangeable indicators [18]. We consider a factorial series distribution for the mixing variable (Section 2.2) and we focus on three particular parametric distributions (binomial, hypergeometric and Stirling). In Section 3, we derive the partial joint distributions of the indicators and their sums. The followed approach, non-traditional, exploits an underlying property of Schur-constancy (Proposition 3). Section 4 provides us with the central result on the dependence structure between the indicators. Working with Kendall's representation, we prove that a convex type order of higher degree on the mixing variable implies a supermodular type order of same degree on the indicators, and conversely (Proposition 4). In Section 5, the convex order condition is discussed for the three mixing distributions of Section 2 when the involved parameter is randomized. We show that a convex type order on the randomized parameter implies a convex type order on the mixing variable (Proposition 5). This part, rather intricated, relies on several preliminary results obtained in Section 7. In particular, we prove that non-negative higher degree increasing convex functions are stable by multiplication (Property 3). Section 6 illustrates some of the results with two applications in insurance and credit risk. For a finite homogeneous portfolio of credit risks, we show that a common shock factor that is less variable implies a weaker strength of dependence between the default indicators (Proposition 9). This result is used to construct higher degree increasing supermodular bounds on the vector of indicators.