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Exact route-length formulas and a storage location assignment heuristic for picker-to-parts warehouses

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ABSTRACT

Order picking is one of the most time-critical processes in warehouses. We focus on the combined effects of routing methods and storage location assignment on process performance. We present exact formulas for the average route length under any storage location assignment for four common routing methods. Properties of optimal solutions are derived that strongly reduce the solution space. Furthermore, we provide a dynamic programming approach that determines storage location assignments, using the route length formulas and optimality properties. Experiments underline the importance of the introduced procedures by revealing storage assignment patterns that have not been described in literature before.

1. Introduction

Warehouses offer a variety of benefits to the company employing them, among which accomplishing least total cost logistics with a desired level of customer service and supporting the just-in-time programs of suppliers and customers (De Koster et al., 2007). However, operating a warehouse can be quite costly as typically a great deal of manual labor is involved. One of the most laborious and costly activities in a warehouse is order picking; the process of retrieving a set of items from storage locations in response to customer orders. The cost of this process can be as much as 55% of the total operating costs of a warehouse (Tompkins et al., 2003). Several operational decisions strongly influence the performance of the order-picking process.

Before customer orders can be retrieved, items must first have been stored in the available locations. The problem of choosing appropriate storage locations for the items is called the storage location assignment problem (Hausman et al., 1976; Gu et al., 2007). This problem shows similarities with the Assignment Problem, a fundamental problem from the field of Operations Research. Both problems require the matching of objects from two mutually exclusive sets, in our case a matching of items to locations. However, the two problems differ strongly in their objective functions. The objective function of the standard Assignment Problem has a simple structure, which makes the problem solvable in polynomial time. The storage location assignment problem, on the other hand, has the objective to minimize the average route length traveled by the workers (order pickers) while retrieving items from locations in the warehouse. This results in a complex function that depends on the layout of the warehouse, the routing method employed, the demand frequencies of all items, and the item-to-location assignment itself.

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The routing method prescribes how an order picker should navigate the warehouse to collect the items in an order. A good routing method yields short route lengths, which decreases the time to pick an order. The physical structure of the warehouse limits the movements of the order picker, who has to navigate between the racks that store the items. Essentially, the routing problem in warehouses classifies as a special case of the Steiner Traveling Salesman Problem (see De Koster et al., 2007), that in some layouts is solvable to optimality in polynomial time (Ratliff and Rosenthal, 1983). However, as De Koster et al. (2007) note, "in practice, the problem of routing order pickers in a warehouse is mainly solved by using heuristics". For this purpose, a number of heuristic routing methods are available from literature. These heuristics can be of a constructive (greedy) nature, employing properties of the layout to find solutions (Hall, 1993) or contain a procedure to search the solution space (Theys et al., 2010).

In this paper, we develop methods for determining storage location assignments in warehouses where order pickers span multiple aisles in each route, and where each order may contain any number of items to be picked (*multi-aisle multi-item picking*). We solve a number of multi-aisle multi-item storage location assignment problems and can guarantee optimality for the first time. As a core component for achieving this, we present formulas for four routing methods that give the value for expected route length in multi-aisle multi-item picking warehouses. In contrast to existing research work, these formulas give the exact expected route length, not an approximation, and they hold for any storage location assignment. Furthermore, structural properties of optimal solutions are derived, which aid in narrowing down the solution space. Due to the low computational requirements for our formulas, in combination with the structural properties, a number of instances can be solved to proven optimality by means of complete enumeration. Using the formulas at its core, we present a dynamic programming approach for determining storage location assignments. The dynamic program provides optimal solutions for one routing method, and near-optimal solutions for another routing method. For the remaining two routing methods, we have no optimal benchmark, but the dynamic program is shown to consistently outperform common storage location assignment rules from literature. Results from our experiments demonstrate patterns for storage location assignment that have not been described before in literature, and that challenge current assumptions about predetermined storage location assignment patterns. A detailed comparison of our work to existing literature is given in Sections 3 and 4.

The remainder of this paper is organized as follows. First, in Section 2 the warehouse layout and the routing methods used are explained. Thereafter, the backgrounds on storage location assignment and on route length estimation are given in Sections 3 and 4, respectively. In Section 5 the problem is defined mathematically. In Section 6 we derive formulas for the exact expected route lengths for four routing methods. These formulas are used to determine optimality conditions in Section 7 and serve as inspiration for the Dynamic Program presented in Section 8. Section 9 provides computational results. Finally, concluding remarks are given in Section 10.

2. Warehouse layout and routing methods

We consider a warehouse with two cross aisles as depicted in Fig. 1. The warehouse has a number of parallel aisles where items are stored in locations. A front cross aisle and a back cross aisle provide access to the aisles, but do not contain storage locations themselves. All routes start and end at the depot (marked "D" in Fig. 1, where the order picker takes an empty

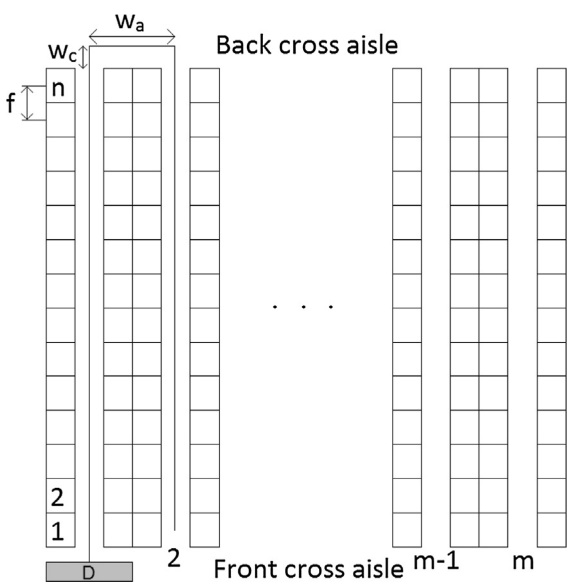

Fig. 1. Schematic overview of the warehouse.