On Drawdown-Modulated Feedback
Control in Stock Trading
Chung-Han Hsieh and B. Ross Barmish

ECE Department
University of Wisconsin
Madison, WI 53706
e-mail: hsieh23@wisc.edu, barmish@engr.wisc.edu

Abstract: Control of drawdown, that is, the control of the drops in wealth over time from peaks to subsequent lows, is of great concern from a risk management perspective. With this motivation in mind, the focal point of this paper is to address the drawdown issue in a stock trading context. Although our analysis can be carried out without reference to control theory, to make the work accessible to this community, we use the language of feedback systems. The takeoff point for the results to follow, which we call the Drawdown Modulation Lemma, characterizes any investment which guarantees that the percentage drawdown is no greater than a prespecified level with probability one. With the aid of this lemma, we introduce a new scheme which we call the drawdown-modulated feedback control. To illustrate the power of the theory, we consider a drawdown-constrained version of the well-known Kelly Optimization Problem which involves maximizing the expected logarithmic growth of the trader’s account value. As the drawdown parameter $d_{\text{max}}$ in our new formulation tends to one, we recover existing results as a special case. This new theory leads to an optimal investment strategy whose application is illustrated via an example with historical stock-price data.

© 2017, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Financial Engineering, Stochastic Systems, Robustness

1. INTRODUCTION

Control of drawdown, that is, the control of the drops in wealth over time from peaks to subsequent lows, is of great concern from a risk management perspective. Suffice it to say, the issue of drawdown control has received a considerable attention in the finance literature. More specifically, to properly control drawdown in a stock trading context, one standard approach is to incorporate this consideration into a constrained optimization framework which typically includes other performance criteria; e.g., see [1] and [2] which focus on a single-stock scenario. There are also some papers dealing with modifications and extensions of these results for the single-stock case to address an entire portfolio under various stochastic modeling assumptions; e.g., see [3]-[8]. Finally, the literature also includes various methodologies to address different types of drawdown. For example, absolute drawdown is studied in [11] and other drawdown-based metrics are considered in [4], [9] and [26].

In this paper, we focus on the notion of percentage drawdown whose technical definition is given in the next section. We provide a new result which enables a trader to guarantee that a prescribed maximum level $d_{\text{max}}$ for this quantity will not be exceeded. To provide further context for this paper, we mention a sampling of some other papers in the existing literature using different risk measures rather than drawdown. Examples of such measures include Value at Risk (VaR), Conditional Value at Risk, Expected Shortfall and the celebrated mean-variance criterion; e.g., see [18]-[20] and [27]. In addition to the analysis of risk, some papers in the literature consider portfolio optimization involving a maximization of expected logarithmic growth. This is the so-called Kelly Optimization Problem which will be used to demonstrate our theory; e.g., see [13]-[15]. Related to this, the literature also includes a well-known method called the Fractional Kelly Strategy. This is aimed at scaling down the size of investment for the purpose of mitigating the risk; e.g., see [12] and [16].

The takeoff point for this paper, which we call the Drawdown Modulation Lemma, characterizes investments which guarantee that the percentage drawdown is no greater than a prespecified level $d_{\text{max}} \in (0, 1)$ with probability one. To make our exposition appealing to this community, this investment scheme which we derive from the lemma, is expressed in a classical feedback control setting. We call it drawdown-modulated feedback control. As the drawdown parameter $d_{\text{max}}$, we recover existing results as a special case. To further illustrate its use, as previously mentioned, we consider a drawdown-constrained version of the Kelly Optimization Problem which involves maximizing the expected logarithmic growth. Then, a numerical example with historical data is carried out and a further generalization of the lemma for portfolio case is discussed in this paper.

2. DRAWDOWN DEFINITIONS

For $k = 0, 1, 2, \ldots, N$, we let $V(k)$ denote the account value at stage $k$. Then as $k$ evolves, the percentage drawdown (to-date) is defined as

$$d(k) = \frac{V_{\text{max}}(k) - V(k)}{V_{\text{max}}(k)}$$

where

$$V_{\text{max}}(k) = \max_{0 \leq i \leq k} V(i).$$

This leads to overall percentage drawdown

$\frac{d_{\text{max}}}{\text{to-date}}$
The percentage drawdown is defined as
\[ d(k) = \frac{V_i - V_k}{V_i}. \]

\[ d_{\text{max}}^* = \max_{0 \leq k \leq N} d(k). \]

Note that the percentage drawdown satisfies \( 0 \leq d(k) \leq 1 \).

Although not considered here, there is another well-known measure, called the maximum absolute drawdown, which we denote by \( D_{\text{max}}^* \) and is given by
\[ D_{\text{max}}^* = \max_{0 \leq k \leq N} V_{\text{max}}(k) - V(k). \]

The reader is referred to [10] and [23] for work on this topic.

To further elaborate on these two notions of drawdown, we consider an example with a hypothetical account value \( V(k) \) shown in Figure 1. From the plot, we obtain the overall percentage drawdown, \( d_{\text{max}}^* \approx 0.8333 \), occurs at stage \( k = 7 \). On the other hand, the maximum absolute drawdown, \( D_{\text{max}}^* = 4.5 \), occurs at stage \( k = 24 \). This toy example shows that the two types of drawdown described above can be quite different. In this paper, we concentrate on the percentage drawdown. This is the version of drawdown which is better suited to deal with different “scales” for \( V(k) \). That is both small and large investors can identify with this concept.

### 3. FEEDBACK CONTROL POINT OF VIEW

In the sequel, as previously mentioned, we emphasize the control-theoretic point of view. Our formulation here is consistent with a growing body of the literature addressing finance problems but originating from the control community; e.g., see [22]-[25]. Although our analysis to follow can be carried out without reference to control theory, to make the work accessible to this audience, we use the language of feedback systems. Specifically, we view the stock prices \( S(k) \) as exogenous inputs to a feedback system. In this setting, we use a linear feedback controller which modifies the investment \( I(k) \) using a time-varying gain \( K(k) \) applied to the account value \( V(k) \). That is, for each stage \( k \), we consider the controller having the form
\[ I(k) = K(k)V(k). \]

Typically, when selecting this feedback gain, we include a constraint which we express as \( K(k) \in \mathcal{K} \). For instance, suppose we restrict attention to a so-called cash-financed investment. Then we impose a constraint which guarantees \( |I(k)| \leq V(k) \).

That is, the investment level is limited to the value of one’s account. This in turn forces \( |K(k)| \leq 1 \). The feedback control configuration which describes this scheme is depicted in Figure 2 for a single stock. Such a configuration can be generalized to deal with a portfolio case of \( n \) stocks; e.g., see [4]. To link back to finance concepts, the special case of buy-and-hold is obtained when \( K(k) \equiv 1 \).

### 4. STOCK-TRADING FORMULATION

For \( k = 0, 1, 2, \ldots, N \), we let \( S(k) > 0 \) denote the stock price. The associated returns up to stage \( N - 1 \) are given by
\[ X(k) = \frac{S(k + 1) - S(k)}{S(k)}. \]

In the sequel, we assume that the returns are bounded; i.e.,
\[ X_{\text{min}} \leq X(k) \leq X_{\text{max}} \]

with \( X_{\text{min}} \) and \( X_{\text{max}} \) being points in the support, denoted by \( \mathcal{X} \), and satisfying
\[ -1 < X_{\text{min}} < 0 < X_{\text{max}}. \]

In the sections to follow, when necessary, we assume the random variables \( X(k) \) are independent and identically distributed (i.i.d.).

**Account Value and Drawdown:** Now letting \( I(k) \) be the investment at stage \( k \) and note that \( I(k) < 0 \) stands for short selling. Beginning at some initial account value \( V(0) > 0 \), the evolution to terminal state \( V(N) \) is described sequentially by the recursion
\[ V(k + 1) = V(k) + I(k)X(k). \]

Now, given a maximum acceptable drawdown level \( d_{\text{max}} \) satisfying \( 0 < d_{\text{max}} < 1 \), we focus on conditions on \( I(k) \) under which satisfaction of the constraint
\[ d(k) \leq d_{\text{max}} \]

is assured along all sample paths with probability one.

**Idealized Market:** In the sequel, we further assume that our stock-trading occurs within “idealized market.” That is, we assume zero transaction costs, zero interest rates and perfect liquidity conditions. For more details about idealized market assumption, the reader is referred to reference [25].
دریافت فوری متن کامل مقاله

امکان دانلود نسخه تمام متن مقالات انگلیسی
امکان دانلود نسخه ترجمه شده مقالات
پذیرش سفارش ترجمه تخصصی
امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
امکان دانلود رایگان ۲ صفحه اول هر مقاله
امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
دانلود فوری مقاله پس از پرداخت آنلاین
پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات