Commun Nonlinear Sci Numer Simulat 43 (2017) 282–295

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Particle-scale modelling of financial price dynamics

David Liu

Department of Mathematical Sciences, Xi'an Jiaotong-Liverpool University, SIP, Suzhou, China 215123

ARTICLE INFO

Article history:
Received 9 February 2016
Revised 19 June 2016
Accepted 15 July 2016
Available online 20 July 2016

Keywords:
Financial Brownian Particle
Order book
Financial market modelling
Discrete particle model
Molecular dynamics

ABSTRACT

This paper proposes a particle-based computational framework for modeling of financial price dynamics, which is an extension of the recent empirical work of Financial Brownian Particle (FBP), and discretizes and solves the Langevin equation that is the continuum representation of a financial market. The framework enables us to simulate the limit order book of the USD/JPY exchange rates. The research yields results that are in good agreement with the published empirical results. Our framework of modelling financial prices is of multidisciplinary nature, and can bridge the fields of empirical studies of financial order books, particle dynamics simulation, and modelling of financial market.

1. Introduction

We propose a particle-based computational framework based upon the empirically developed theory of Financial Brownian Particle (FBP) by Yura et al. [47] to simulate order trading in financial market. This methodology enables us to simulate financial prices such as the USD/JPY exchange rates with consideration of different levels of supply and demand imbalance. It also allows us to examine important issues such as the influence of bid-ask spread on prices, the concept of market drag coefficient, the lifetime distributions of market orders and the proposed micro-force models for price dynamics.

Dynamics of financial price can be interpreted by the continuum representation using the Langevin equation as developed by Yura et al. [47] very recently in their empirical study of the order book of USD/JPY exchange rates. Our contribution is to extend such findings and develop an innovative and feasible model based on the techniques of particle dynamics for solving the Langevin equation for market price dynamics modelling.

Simulations of asset prices adopting Brownian motion of microscopic particles, sometimes referred to as the Wiener process, has a rather long history. Brownian motion can be viewed as a “random walk” process, and was first observed by Robert Brown [7]. The motion of particles stem from the constant forces exerted on the particles from the surrounding fluid molecules bumping into the particles as claimed by Albert Einstein [13]. The actual development of Brownian motion as a stochastic process was made by Norbert Weiner [45], who established the modern mathematical framework of what is known today as the Brownian motion random process or the Wiener process. In modeling asset prices, Brownian motion was mathematically defined by Bachelier [1], who proposed it as a model for asset price movements. However, most current asset price models use the generalized Wiener process (or geometric Brownian motion, or GBM) (See, for example, Hull [29]) to extend the notion of a standard normally distributed variable to a process that evolves over time. The lognormal model is generally used to model stock prices (at least as the basis for the Black-Scholes option pricing model by Black and Scholes [4]) which assumes that over each time interval, the log return in stock prices is normally distributed with constant

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E-mail address: david.Liu@xjtlu.edu.cn

http://dx.doi.org/10.1016/j.cnsns.2016.07.011

mean and standard deviation. However, GBM is not a completely realistic model, in particular it falls short of reality in the following points: In real stock prices, volatility changes over time, and returns are usually not normally distributed. In addition, returns have negative skewness as documented in Wilmott [46]. Empirical evidence of asset returns far from having symmetric distributions suggests alternative models than the GBM, for example, underlying asset price models developed by Huang and Wu [28], Rubinstein [39], and Hutchinson et al. [30]. Nevertheless, the financial economic literature uses the Wiener process as the standard starting point for modeling and financial engineering applications (See, for example, Merton [35]).

The above mentioned price models, however, are only focused on price itself, and on the complicated modifications to the basic random walk model in order to account for the numerous stylized facts of asset prices. There are also other approaches which simulate financial markets and prices by computational economic models with different classes of agent strategies to create small-scale or 'toy' markets played by agents. Various assumptions and complicated trading algorithms usually need to be implemented in order to investigate the actual complex financial markets (e.g. Suominen [42], Hollifield et al. [26], Goettler et al. [21], Roşu [38], Bartolozzi [2]). However, the equilibrium strategies adopted in the models may deviate from the intrinsic mechanism of actual markets. It therefore requires profound research on the 'mirco-dynamics' of financial prices in order to understand how prices change in response to order impacts, and what causes financial price fluctuations. Our understanding on price dynamics has been enhanced by Hasbrouck [24], Hausman et al. [25], Kempf and Korn [32], Evans and Lyons [14], Hopman [27], Plerou et al. [36], Rosenow [37], Bouchaud et al. [5,6], Gabaix et al. [19], Lillo et al. [33,34], Weber and Rosenow [44], Farmer et al. [15–17], and Cont et al. [9], etc. Their work are essentially focused on the equilibrium problems of prices, which are studied by using the price impact functions.

Recently, Yura et al. [47] proposed that the observed financial motion can be analogous to a genuine colloidal Brownian particle embedded in a fluid of smaller particles, which are termed as 'Financial Brownian particle in order-book molecular matter antimatter fluid'. Their empirical work demonstrates that the equilibrium in financial market can be described by the Langevin equation. Our work is to extend their approach, and develop a feasible model for modelling of financial prices computationally. Intrinsically, our simulations of markets are not only based upon particle dynamics, but also upon solving the proposed market equilibrium equation (i.e. the Langevin equation) rather than an artificially designed market equilibrium strategy.

The simulations in this paper show that the price dynamics of USD/JPY can be modelled sensibly by the proposed method, and many important influential factors can be examined. For instance, we show that the effect of adjusting bid-ask spreads on the trading is that increasing the bid-ask spread will lead to the reduced transaction rates (i.e. number of transactions per unit time). We also introduce the concept of market drag coefficient, and study its influence on prices. By varying the drag coefficient, both sub-diffusive and super-diffusive prices can be observed.

In addition, our research proposes to use the so-called 'micro-force' models for the price response to order impact, which works effectively. Meanwhile, the simulated results of the lifetimes of orders are in good agreement with the published empirical results by Yura et al. [47]. Our research would open up a new direction for particle-based modelling of financial prices.

2. Framework of particle dynamics modelling of financial prices

2.1. The FBP

The concept of the FBP that Yura et al. [47] proposed is illustrated in Fig. 1. The order book of USD/JPY exchange rates analyzed by the researchers is evolved by the spontaneous injection of three types of orders: limit orders, market orders, and cancellation. In the order-book data for currency pairs, any orders, either buy or sell, are quantized by a unit of one million U.S. dollars with the price given with a granularity of 0.0001 yen (called a pip) recorded with a time stamp of one millisecond. A pair of buy and sell orders meeting at the same price immediately triggers a transaction and disappear from the order book just like a pair annihilation of matter-antimatter. The price and time quantization enable the researchers to describe the market by particles in discrete space and time, where a particle represents either a buy or sell order of one million U.S. dollars. They assign a superscript $-(+)$ for buy (sell) orders. At a given time, a state of the market is characterized by its order book contains the set of yet unrealized buy (sell) orders in the lower (higher) side of the discrete price axis. The highest buy (lowest sell) order price, denoted as $x^-(t)(x^+(t))$, is called the best bid (ask), and the gap between the best bid and best ask is called the spread. For each buy (sell) order in the order book, they introduce an important measure of depth $\gamma^-(\gamma^+)$, which is defined by the distance of this buy (sell) order from $x^-(t)(x^+(t))$ in pip units.

As shown in Fig. 1, an imaginary colloidal Brownian particle, called a colloid, has its center positioned at the mid-price, $x(t) = \{x^-(t) + x^+(t)\}/2$, with the core diameter given by the spread $x^+(t) - x^-(t)$. The accumulated orders are regarded as embedding fluid particles with a diameter equal to 1 pip. The core of the colloid (the yellow disk) and the interaction range (the blue ring area) overlap with the particles near the spread (the green disk). This interaction range is called the inner layer and the domain outside of this interaction range the outer layer. The values of threshold depths for defining the inner layer, $\gamma_c^-(\gamma_c^+)$ can be estimated from the order-book data. With the injection of new orders, the surrounding particles change their configuration and the colloid moves as a result. Yura et al. [47] have further demonstrated that the velocity of