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Chaotic dynamics in nonlinear duopoly Stackelberg game with heterogeneous players

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HIGHLIGHTS

- A nonlinear duopoly Stackelberg model of competition on output between heterogeneous players is investigated.
- For the duopoly Stackelberg model, one player is bounded rationality and another is adaptable expectation.
- Complex dynamic behaviors would occur while varying the values of some main parameters.
- Appropriate controlling parameter can be chosen to force the chaotic system back to stability.
- Fractal dimension of this system is studied.

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ABSTRACT

In this paper, a nonlinear duopoly Stackelberg game of competition on output is concerned. In consideration of the effects of difference between plan products and actual products, the two heterogeneous players always adopt suitable strategies which can improve their benefits most. In general, status of each firm is unequal. As the firms take strategies sequentially and produce simultaneously, complex behaviors are brought about. Numerical simulation presents period doubling bifurcation, maximal Lyapunov exponent and chaos. Moreover, an appropriate method of chaos controlling is applied and fractal dimension is analyzed as well.

1. Introduction

In production practice, firms usually produce multi-products rather than single type to obtain maximum profits [1]. However, for the sake of research, the competition in homogeneous market has been often chosen to study the dynamical behaviors. The structure that a homogeneous market is almost completely controlled by a tiny number of firms exists generally. For a long time, the conflict of interests between two players in duopoly game has been researched widely. The earliest mathematical model of competition in oligopoly market is proposed by French economist, Antoine Augustin Cournot in 1838 [2]. The model described how the firms' production decisions influence each other while they did not coordinate in producing. It is also the earliest version of the Nash equilibrium application [3–5]. Gradually, Cournot model became a starting point for the analysis of oligopoly theory. After nearly one hundred years, H. von Stackelberg enriched this theory and formed Stackelberg model, which contains two players named leader and follower. The two firms need take both the consumers' behavior and the competitor's reaction into consideration. To a certain extent, this situation would

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lead the dynamic of oligopoly model to be much more complex. Refs. [6] studied an oligopoly consisting of M leaders and N followers that supply a homogeneous product (or service) non-cooperatively. Refs. [7–10] analyzed the dynamical evolution of a bounded rationality duopoly game with delay. The classical Stackelberg game has been extended to bounded rational price Stackelberg game in Refs. [11]. Refs. [12–14] described several other important applications of bounded rationality. Refs. [15] investigated the dynamics of a Cournot duopoly game with differentiated goods in which bounded rational firms apply a gradient adjustment mechanism to update the quantity produced in each period. Dynamics behaviors of two-delays connections has been presented in Refs. [16–18]. Refs. [19] showed a new four-dimensional hyperchaotic system developed by extension of the generalized diffusionless Lorenz equations. Two different time delay structures for the dynamical Cournot game with two heterogeneous players are considered in Refs. [20–22]. Refs. [23] formulated a price competition model with two heterogeneous players participating in carbon emission trading. In addition, some other situations based on the Cournot model and Stackelberg model have been studied in Refs. [24–28].

Linear difference equations with constant coefficients can describe linear time invariant discrete systems [29]. Yet in order to close the reality system more, more often than not, we study nonlinear dynamics. Referring to the previous works, there are fewer existing literature based on Stackelberg model than Cournot model to discuss the complex dynamics. Nevertheless, differences between the two players in duopoly model exist in practical life obviously. The Stackelberg game has been proved to be the most appropriate model for many applications which involve human interaction [30]. For the Stackelberg game, the leader firm has the ability to promise and possesses absolute advantage of moving first. In other words, the position between competing firms is not symmetrical.

The purpose of this paper is to formulate a duopoly Stackelberg model with heterogeneous players which can make up the deficiencies mentioned above, simulate complex dynamical behaviors of the system and achieve chaos control. Both the leader firm and the follower firm try to maximize their profits according to local information of the strategies. Because of the difference in status, the two players put forward different strategies to deduce expected output. The leader represents a bounded rational player and the follower has adaptable expectation.

This paper consists of five parts. In Section 2, the duopoly Stackelberg model is briefly introduced and a two-dimensional system with heterogeneous players is formulated. In addition, we discuss the equilibrium points and local stability of the game. In Section 3, the main numerical simulations are demonstrated. As the chaotic state emerging during the simulation process, a chaos control model is presented to put it right. In Section 4, the fractal dimension of strange attractor in this system is estimated numerically. At last, we summarize some important conclusions of this work.

2. The nonlinear duopoly Stackelberg game with heterogeneous players

Stackelberg formulated a model of yield leadership which was applicable for homogeneous market. In the order of action, there is a difference between the manufacturers. Production decisions are based on the following sequence: the leader firm announces forward production first according to the *rule of thumb*, then the follower observes his action and makes production decision accordingly. So in planning phase, the leader must consider how the follower will react, which means the leader firm knows response function of the follower and can predict the output impact of follower roughly. Under this situation, the leader's decision on yield is a profit maximizing yield which is bound to react the follower's response function. That is, in the Stackelberg model, there is no need for the leader to have his own response function.

The duopoly Stackelberg model is divided into two stages. Stage 1 is called *planning phase* and stage 2 is called *producing phase*. In the first stage, each firm chooses strategies and concludes forward output contracts sequentially. Then in the second stage, they produce simultaneously.

In this model, we consider two firms in the same oligopoly market with nonlinear cost function

$$C_i(q_i) = c_i[q_i(t) - Q_i(t)]^2, \;\; i = 1, 2 \tag{1}$$

where i represents firm serial number, i.e., firm 1 is the Stackelberg leader and firm 2 is the follower. $q_i(t)$ represents the output of firm i during discrete period $t = 0, 1, 2 \ldots$ with the production cost function $C_i(q_i)$, obviously, $q_i(t) > 0$. $Q_i(t)$ stands for the announced plan quantities of firm i respectively and c_i are the positive shift parameters to the cost function of firm i. In fact, there are many factors affecting the diversities of ideal production and actual production. For the sake of the research in this paper, we summarize all of these abstract factors as the cost coefficient c_i. The selling price at period t is determined by the two firms total supply

$$q(t) = q_1(t) + q_2(t) \tag{2}$$

through a linear demand function

$$p = a - bq \tag{3}$$

where a and b are positive constants. Based on the above assumptions, the single-period profit of firm i is given by

$$\Pi_i(q_1, q_2) = q_i(a - bq) - c_i(q_i - Q_i)^2, \;\; i = 1, 2 \tag{4}$$