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Price competition in a nonlinear differentiated duopoly

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ABSTRACT

This article represents an attempt to characterise the dynamics of a nonlinear duopoly with price competition and horizontal product differentiation by accounting for non-negativity constraints (on profits and the market demand). The model is set up by following the tradition led by Bischi et al. (1998), according to which players have limited information. The article shows several local and global phenomena of a two-dimensional discrete time system when the price demand elasticity varies. It also points out the differences from both a mathematical and economic point of views in terms of dynamic outcomes when the non-negativity constraints are not binding and when they are binding. This is done by combining mathematical techniques and simulation exercises.

1. Introduction

This article represents an attempt to characterise local and global dynamics in a nonlinear duopoly with price competition and horizontal differentiation. Particular attention is paid to the role played by non-negativity constraints on quantities and profits in determining long-term outcomes.

In both static and dynamic models of industrial economics and industrial organisation, scholars often avoid dealing with microeconomic settings where the marginal willingness to pay of consumers is captured by non-linear inverse demand functions. The majority of contributions, in fact, was concerned with models characterised by linear demands. This assumption is essentially made up for analytical tractability as well as for preserving concavity in the firm's optimisation problem and the uniqueness of the equilibrium. However, from an empirical point of view, there exist robust estimations supporting the existence of (individual or market) inverse demand functions describing a non-linear course between price and quantity [25,26] This holds in models with both price competition (Bertrand) and quantity competition (Cournot). We recall that when products are homogeneous firms choose to compete on quantities [22]. In fact, a classic result in the duopoly theory is that price competition with homogeneous products leads to the unrealistic Bertrand paradox, resulting in the zero profit condition for competitors. This result does no longer hold in the case of product differentiation. In fact, as noted by Puu [21], in the latter case the two firms have a kind of monopoly power (monopolistic competition) and hence a price competition setting can also be compatible with the existence of a Nash equilibrium with positive profits. Even empirically, there are classic results that appear to confirm that on actual markets competition takes place on prices when products are differentiated [17].

Given the importance of studying problems related to product differentiation,[1] our goal is to analyse the dynamic properties of a two-dimensional discrete time model where firms compete on prices. As obtaining and using information efficiently (as predicted by the rational expectations paradigm) seems to be a too strong assumption which can lead to high costs [8], we assume that players have limited information and use a naïve rule (bounded rationality) to set the price for the subsequent period.[2] Roughly speaking, when bounded rationality is introduced in economic models results are somewhat different from those obtained under rational expectations, especially with regard to stability outcomes. In fact,

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[1] See [22] for the case of profit-maximising firms and [15] for the case of managerial firms.

[2] Models with rational expectations are based on two main assumptions: rationality of agents (i.e., expectations of an economic agent are equivalent to mathematical expectations, which exploit all available information) and homogeneity of expectations of all economic agents.

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in the absence of full information models predict that instability holds under weak conditions[3]: the instability of equilibria seems to be the rule rather than the exception and may represent an explanation of the observed output fluctuations (quantity or price) in imperfectly competitive markets.

With specific regard to the duopoly dynamics, the literature has essentially concentrated on quantity-setting firms by assuming either linear demand (quadratic utility) or unit-elastic demand (Cobb–Douglas utility). In a leading article, Bischi et al. [6] draw attention on the existence of heterogeneities between players (asymmetric map) that give rise to symmetry-breaking bifurcations. They show that negligible differences in the basic parameters characterising duopolistic firms may cause qualitatively different dynamic evolutions as compared with the case of a symmetric game (such as lack of synchronisation and coexistence of attractors), i.e. the representative agent hypothesis dramatically matters for the outcome the economy may follow. More recently, it has been pointed out that there may exist similar dynamic events also when players are homogeneous and then the system characterising the dynamics of the economy is symmetric [12,14]. Specifically, Fanti et al. [12] develop a nonlinear duopoly model with linear demand and managerial firms to show that the degree of competition between managers is responsible for on-off intermittency, blow-out phenomena and multistability, whereas Fanti et al. [14] consider a model with homogeneous products, general isoelastic demand and profit maximising firms finding that the elasticity of market demand is responsible for local and global outcomes that cannot be observed in the case of unit-elastic demand (coexistence of attractors, coordination failures and complex structures of the basins of attraction). In this literature, some exceptions that study price competition with limited information are the works of [3,4,16]. The first work considers a horizontally differentiated duopoly with homogeneous firms and linear demand (symmetric map). The main aim is to show that the extent of product differentiation is responsible for synchronised dynamics along the invariant diagonal and intermittency. The work also discusses the transition from simple dynamics to complex dynamics and describes the structure of the attractors and their basins. The second article concentrates on the case of a differentiated duopoly with substitute goods where consumers' demands are defined by starting from a CES (Constant Elasticity of Substitution) utility function, showing how different parameter configurations can affect the dynamics of the system. The third work, instead, considers a vertically differentiated duopoly and compares local and global dynamics of Cournot and Bertrand competition models. However, and most importantly, these articles do not take into account non-negativity constraints on quantities and profits and their role in long-term outcomes.

The present work emphasises the importance of economic constraints in determining dynamic outcomes when price competing firms do not have complete information. Results are given by considering gradual reductions in a parameter that contributes to measure the demand elasticity. This reduction may produce an increase in both unitary profits and marginal profits that contributes to let the change of prices over time much more reactive (to capture the opportunity of higher profitability) than when the elasticity of market demand is lower. An increase in the reactivity of firms in setting the price between two subsequent periods is responsible for several phenomena (Neimark–Sacker bifurcations, multistability) that are impossible in the case of unit-price elasticity. With regard to the role of economic constraints, the introduction of a piecewise map that makes it feasible regions of the phase plan that were excluded from previous studies leads to changes in the dynamics properties of the system that have required a thoughtful analysis. In particular, new forms of multistability and synchronisation phenomena not observable in the absence of constraints may arise. From an economic point of view, it is possible to observe time series characterised by periods in which production is positive spaced out by periods without production or periods in which production is positive but leading to a very low mark up of the price per unit of good with respect to the marginal cost. This behaviour seems to mimic some of the phenomena observed in actual duopolies, as price competition is fiercer than quantity competition [11,19].

In the literature, there are contributions that have introduced constraints on both the demand side and the supply side (no zero production) in nonlinear duopoly models. As far as the former group of works is concerned, we mention the article of Bischi and Lamantia [5] showing, in the basic Cournot model with linear reaction functions, that the existence of constraints on the size of market demand can generate routes to complex dynamics by starting from a situation where the Nash equilibrium is positive but unstable in the absence of constraints. As far as the second group of works is concerned, the literature [2,24] focuses on models with quantity competition and perfect knowledge of the market demand (best reply). In particular, Agliari et al. [2] emphasise the possibility of obtaining multistability (attractors coexisting with a stable Nash equilibrium) due to constraints on the supply side of the market. Finally, with the aim of avoiding the unpleasant result that once a firm chooses to do not produce it definitely exits the market, Tramontana et al. [24] introduce a lower bound on the amount produced by each single firm. Our price-setting mechanism resembles this assumption. However, different from [24] in the present work it is possible to obtain that the quantity demanded by customers corresponding to a certain price at some time is null (with the opportunity to re-enter to the market later). This is because competition (firms' decisions) takes place on prices and market demand is the result of such decisions.

The rest of the article is organised as follows. Section 2 develops a Bertrand game with horizontal product differentiation in a standard static set up. Section 3 introduces a dynamic mechanism of prices by assuming limited information as in [7] and shows that reducing (ceteris paribus) the demand elasticity causes several local and global phenomena (Neimark–Sacker bifurcation, multistability and so on) that cannot be observed in a model with unit-elastic demand. Section 4 explores the role of economic non-negativity constraints by adopting the critical curves technique. Section 5 outlines the conclusions.

2. The game

Consider a Bertrand duopoly with horizontal product differentiation and two types of agents: firms and consumers. Firm 1 produces output of variety 1 ($q_1 \geq 0$) and firm 2 produces output of variety 2 ($q_2 \geq 0$). We assume that the indirect demand of product $i = 1, 2$ takes the following form:

$$p_i = Q_i^{\frac{-1}{\eta}}, \tag{1}$$

where $Q_i = q_i + dq_{-i}$, $-i$ identifies the rival firm of i, $\eta > 0$ is a parameter that contributes to measure the degree of demand elasticity, $p_i \geq 0$ is the consumers' marginal willingness to pay towards product of variety i produced by firm i and d is the degree of horizontal product differentiation. The demand function in (1) modifies the general isoelastic demand function of [14] by considering horizontally differentiated products. The direct demand of product of

[3] Rational expectations models tend to explain the volatility of economic variables because of the existence of exogenous (stochastic) shocks, while instability in models with bounded rationality is endogenous to the model. See [1,2] for a critique to the use of rational expectations in dynamic (nonlinear) models.