Forest valuation under the generalized Faustmann formula with taxation

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A R T I C L E   I N F O

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A B S T R A C T

This article presents the second part of a two-part series on forest taxation and valuation. In this second part, I derive the forest valuation formulas under the generalized Faustmann formula with five types of forest property tax. They are the unmodified property tax, the site value tax, the flat property tax, as well as the gross and net forest productivity taxes. Under the assumption of backward tax shifting, all of the taxes result in lower land expectation values. If the tax does not affect the optimal harvest age empirically, then the value of the standing timber remains the same with or without taxation. When a forest property tax, such as an unmodified property tax, shortens the optimal harvest age, it will also affect the value of the standing timber.

1. Introduction

Recently, Chang (2014) published the formula for forest valuation under the generalized Faustmann formula without forest taxation. Given that some type of forest property tax is imposed in all 50 states in the United States and most countries around the world, this paper extends the formula for forest valuation to include these forest property taxes. These formulas are needed to help forestland owners and managers determine properly the value of the land as well as the standing timber and provide legislators with information on how the various types of forest property taxation may affect the value of the forestland and the standing timber differently. As Chang (2014) pointed out, improper valuation of the land value could lead to under-utilization of the standing timber by as much as 30%, resulting in over-payment of income tax at the time of timber harvest. Furthermore, as states, such as Maine, contemplate property tax reform, the magnitude of the impact of different types of forest property tax on forest valuation can be highly relevant.

Following Chang’s (1982) classification, forest property taxes can be separated into unmodified property tax (UPT), site value tax (SVT), flat property tax (FPT) and forest productivity tax including the gross forest revenue productivity tax (GFRPT) and the net forest revenue productivity tax (NFRPT). In the United States, for example, Kentucky imposes an unmodified property tax, Michigan imposes a flat property tax, Texas imposes a gross forest revenue productivity tax, and Louisiana imposes a net forest revenue productivity tax.

Unmodified property tax, as the original form of property tax, imposes a tax on both the value of the land and the standing timber every year. In practice, the same tax rate applies to both these values. However, to obtain the general form of its formula, during its derivation an $x_1 \%$ tax is imposed on the value of the land while a $y_1 \%$ tax is imposed on the value of the standing timber per acre or hectare annually. Under such a framework, site value tax represents a special case of the unmodified property tax when only the value of the land is taxed at $x_1 \%$ annually. Its formula, therefore, can be obtained as a special case from the general formula for the unmodified property tax. Flat property tax (FPT) imposes a fixed amount of tax $X_1$ per acre or hectare regardless of its value. Two types of forest productivity tax are in use – a gross forest revenue productivity tax (GFRPT) based on the gross average revenue per year and a net forest revenue productivity tax (NFRPT) based on the net average revenue per year by deducting an annual management cost from the gross average revenue. Once the amount of the productivity tax is determined under either one, they can be inserted into the valuation formula for the flat property tax to obtain their respective formulas.

1.1. Formula derivations

In this section the relevant formulas of forest valuation under various forms of forest property taxation are derived and presented together. Before their derivations, it should be noted that similar to the forest taxation part of this series, all of the taxes are assumed to shift backward into lower land value. Below, the case of the generalized Faustmann formula without taxation is presented first.

1.1.1. Forest valuation without taxation

As shown in Chang (2014),...
\[
\text{LEV}_{1\text{ no tax}} = \left[ V_1(t_1) + \sum_{i=1}^{n} A_{\text{land}} e^{r_1(t_i-t_{i-1})} - C_{t} e^{r_1 t_1} + (\text{LEV}_2 - \text{LEV}_{1\text{ no tax}}) \right] (e^{r_1 t_1} - 1)^{-1}
\]  

(1)

or

\[
\text{LEV}_{1\text{ no tax}} = \left[ V_1(t_1) + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} - C_{t} e^{r_1 t_1} + \text{LEV}_2 \right] e^{-r_1 t_1} \tag{1a}
\]

And the formulas for forest valuation under the generalized Faustmann formula without tax are

\[
FV_{1}(s_1)_{\text{no tax}} = \left[ V_1(t_1) + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} + \text{LEV}_2 \right] e^{-r_1 t_1} \tag{2}
\]

\[
L_{1}(s_1)_{\text{no tax}} = \left[ \text{LEV}_{1\text{ no tax}}(e^{r_1 t_1} - e^{r_1 s_1}) + \text{LEV}_2(e^{r_1 s_1} - 1) \right] (e^{r_1 s_1} - 1)^{-1} \tag{3}
\]

Essentially, \( L_1(s_1) \) is a weighted average, modified by the discounting process, of the value of the land at the beginning (LEV\(_1\)) and the end (LEV\(_2\)) of the first rotation. Interested readers are encouraged to consult Chang (2014) for its derivation.

\[
\hat{V}_1(s_1)_{\text{no tax}} = FV_{1}(s_1)_{\text{no tax}} - L_{1}(s_1)_{\text{no tax}} \tag{4}
\]

for \( 0 \leq s_1 \leq t_1 \) and \( \text{LEV}_2 \) is the land expectation value at the beginning of the second harvest period, \( r_1 \) is the interest rate of the first harvest period, \( V_1(t_1) \) is the stumpage value (final harvest) value of the stand of the first period, \( A_{\text{land}} \) is miscellaneous yearly revenues and expenses during the first period, \( C_t \) is the regeneration cost for the first period, and \( t_1 \) is the harvest age.

\[
FV_{1}(s_1) = \text{the value of a forest with } s_1\text{-year old standing trees and the land.}
\]

\[
L_{1}(s_1) = \text{the value of the land with } s_1\text{-year old standing trees.}
\]

\[
\hat{V}_1(s_1) = \text{the value of the } s_1\text{-year old standing trees.}
\]

It should be noted here that \( FV_{1}(s_1) \) represents the harvest value of a \( s_1\)-year old stand while \( \hat{V}_1(s_1) \) represents the value of a \( s_1\)-year old stand left standing. It must therefore be true that when trees are left standing, \( \hat{V}_1(s_1) \) is greater than \( V_1(s_1) \) its immediate harvest value – stumpage value. Otherwise, the \( s_1\)-year old stand should have been harvested. As shown below, only at age \( t_1 \), the time of the timber harvest would \( \hat{V}_1(t_1) \) be \( V_1(t_1) \).

Note that when \( s_1 = t_1 \), from Eq. (3) \( L_{1}(t_1) = \text{LEV}_2 \)

\[
\hat{V}_1(t_1) = FV_{1}(t_1) - L_{1}(t_1) = [V_1(t_1) + \text{LEV}_2] - \text{LEV}_2 = V_1(t_1) \tag{5}
\]

when \( s_1 = 0 \), from Eq. (3) \( L_{1}(0) = \text{LEV}_1 \)

\[
\hat{V}_1(0) = FV_{1}(0) - \text{LEV}_1 = \left[ V_1(t_1) + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} + \text{LEV}_2 \right] e^{-r_1 t_1} - \text{LEV}_1
\]

\[
= \left[ V_1(t_1) + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} - C_{t} e^{r_1 t_1} + \text{LEV}_2 \right] e^{-r_1 t_1} + C_{t} e^{r_1 t_1} e^{-r_1 t_1}
\]

\[
= \text{LEV}_1 + C_{t} - \text{LEV}_2 = C_{t}
\]

Thus, the two values at the harvest age \( t_1 \) and age 0 are known before any calculations. They, therefore, help verify if empirical calculations are done correctly. Presented in Table 1 are \( \text{LEV} \), \( \text{LEV}_1 \), \( FV(s) \), \( L(s) \) and \( \hat{V}(s) \) without taxation to provide the basis for comparisons with those under various types of forest property taxation.

1.1.2. Forest valuation under the unmodified property tax (UPT)

Under the unmodified property tax, as shown in Chang (2018)

\[
\text{LEV}_{\text{UPT}} = \left[ \frac{(n_1 + y_1)}{(n_1 + x_1)} \right] \left[ V_1(t_1) + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} - C_{t} e^{r_1 t_1} + (\text{LEV}_2 - \text{LEV}_{1\text{ UPT}}) \right] (e^{r_1 t_1} - 1) \tag{7}
\]

or

\[
\text{LEV}_{1\text{ UPT}} = \frac{(n_1 + y_1)}{(n_1 + x_1)} \left[ V_1(t_1) + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} - C_{t} e^{r_1 t_1} + \text{LEV}_2 \right] \tag{7a}
\]

where \( \text{LEV}_{1\text{ UPT}} \) is the land expectation value under the unmodified property tax, as shown in Chang (2018)

\[
FV_{1}(s_1) = \text{at age } s_1, 0 \leq s_1 \leq t_1 \text{ would have been}
\]

\[
FV_{1}(s_1)_{\text{UPT}} = \left[ V_1(t_1) + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} + \text{LEV}_2 \right] e^{r_1 t_1} \tag{8}
\]

if the value of the land and the standing timber were taxed at the same rate \( y_1 \). When the land is taxed at the rate of \( x_1 \) different from \( y_1 \), the difference \( (y_1-x_1)\text{LEV}_1 \) must be added back to the forest value above. As such,

\[
FV_{1}(s_1)_{\text{UPT}} = \frac{\text{LEV}_2 + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} + V_1(t_1)}{e^{r_1 t_1}}
\]

\[
+ \frac{(y_1 - x_1)\text{LEV}_1}{(n_1 + x_1)} \int_{s_1}^{t_1} e^{r_1(t_1-t)} dj \tag{9}
\]

Since the effective interest rate is the sum of the interest rate plus the tax rate on the value of the stand, the value of the land with \( s\)-year-old trees \( L_{s}(s_1) \), \( 0 \leq s_1 \leq t_1 \) can be calculated as

\[
L_{1}(s_1)_{\text{UPT}} = \left[ \text{LEV}_{1\text{ UPT}}(e^{r_1(t_1-s_1)} - e^{r_1(s_1-t_1)}) + \text{LEV}_2(e^{r_1(s_1-t_1)} - 1) \right] (e^{r_1 t_1} - 1) \tag{10}
\]

The value of the \( s_1\)-year-old standing timber with unmodified property tax can be calculated as

\[
\hat{V}_{1}(s_1)_{\text{UPT}} = FV_{1}(s_1)_{\text{UPT}} - L_{1}(s_1)_{\text{UPT}} \tag{11}
\]

When the tax rate on the value of the land \( x_1 \) and that on standing timber \( y_1 \) are the same, Eq. (9) becomes

\[
FV_{1} = \frac{\text{LEV}_2 + \sum_{m=1}^{i} A_{\text{land}} e^{r_1(t_1-m)} + V_1(t_1)}{e^{r_1 t_1}} \tag{12}
\]

1.1.3. Forest valuation under the site value tax (SVT)

When only the value of the land is taxed under the site value tax \( y_1 = 0 \). As a special case of Eqs. (7), (7a), (9), and (10), respectively
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