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Dynamic provision of public goods under uncertainty☆

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ABSTRACT

This study investigates the private provision of public goods under uncertainty using a general dynamic equilibrium model with stochastic disturbances. In particular, the model incorporates income shocks governed by a Wiener process with a mean of zero and standard deviation of unity as uncertainty. We analyze how uncertainty and population size affect the supply of public goods. Dynamic analysis shows the importance of attitude toward risk and a contrast between short-run and long-run responses to increases in uncertainty and population size. Results show that under specified conditions, escalating uncertainty reduces the long-run contributions to public goods through the stochastic accumulation of capital but it raises short-run contributions. The average contribution increases to a positive finite value by increasing the population to a certain level, but it declines toward zero if the population size is infinite. These twin results are based on the dynamic behaviors of risk-averse individuals responding to elevated risks.

1. Introduction

The private provision of public goods has attracted theoretical, empirical, and experimental studies in public economics. The literature raises numerous implications about the effect of income redistribution and population size on individual contributions to public goods. Theoretical studies discuss income redistribution as a transfer neutrality (Shibata 1971; Warr 1983; Bergstrom et al. 1986; Andreoni 1988, 1989, 1990; Gradstein et al. 1993)[1] and population size in relation to the free-rider problem that causes the inefficient provision of public goods (Olson 1965; Chamberlin 1974; McGuire 1974; Andreoni 1988; Pecorino 1999; Gaube 2001; Kawachi and Ogawa 2006).[2]

Previous theoretical predictions seem inconsistent with available empirical evidence and experimental results[3] and typically are explained by altruism or idiosyncratic behavior under uncertainty. In particular, uncertainty might affect contributions when people base decisions on conjecture. Austen-Smith (1980) incorporates inadequate information about the supply of public goods into his model and argues that a concave utility function representing risk-aversion increases contributions. Succeeding studies involving generally extended models show that the third-order differential term of a utility function significantly affects contributions (Sandler et al. 1987; Gradstein et al. 1993).[4]

In contrast, Eichberger and Kelsey (2002) provide disparate analyses concerning the provision of public goods under uncertainty. They model individuals' beliefs by capacities as ambiguity where it is difficult to assign precise probabilities and investigate how ambiguity influences contributions to public goods. They show that elevated uncertainty raises contributions if the utility function is strictly concave. These models postulate static settings for investigating contributions.

In daily life, people make intertemporal decisions under uncertainty, and some studies examine the dynamic provision of public

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[1] The well-known transfer neutrality theorem argues that an income transfer policy does not affect allocation of goods under voluntary provision of public goods. Shibata (1971) suggested the neutrality result before Warr (1983). Bergstrom et al. (1986) and Andreoni (1988) demonstrate it in a generalized model of voluntary provision of public goods. Andreoni (1989, 1990) employ a model with impure altruism and show that transfer neutrality is broken under impure altruism. Gradstein et al. (1994) study this issue under incomplete information. Uler (2009) investigates the relation between voluntary provision and income redistribution by incorporating redistributive taxation into the model of Bergstrom et al. (1986).

[2] Olson's (1965) pioneering study concerns the free-rider problem. Chamberlin (1974) and McGuire (1974) study how group size affects contributions to public goods, and Andreoni (1988) generalizes their results. Conventional results are that total contributions rise to a finite value and average contributions diminish to zero (Andreoni 1988). Gaube (2001) provides sufficient conditions for those results. Pecorino (1999) and Kawachi and Ogawa (2006) study this issue under repeated game settings.

[3] See Brunner (1998) for empirical evidence from a study of public radio. See Ledyard (1995) for a survey of early experimental studies and Chaudhuri (2011) for a survey of the literature after Ledyard (1995).

[4] Noting an error in Austen-Smith (1980), Sandler et al. (1987) find that risk aversion is insufficient grounds for individuals to increase contributions to public goods.

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goods, albeit without considering stochastic disturbances (Fershtman and Nitzan 1991; Wirl 1996; Itaya and Shimomura 2001; Yanase 2006). Fershtman and Nitzan (1991) study the voluntary provision of public goods in a model wherein contributions accumulate over time. Wirl (1996) extends the model of Fershtman and Nitzan (1991) by incorporating nonlinear strategies. Itaya and Shimomura (2001) present a dynamic conjectural variations model in the private provision of public goods and provide a reasonable micro-foundation for conjectural variations equilibrium. Yanase (2006) presents a generally extended model of Itaya and Shimomura (2001). Recently, Wang and Ewald (2010) extended Fershtman and Nitzan (1991) in a stochastic model.

These authors study accumulation of contributions and show that the equilibrium provision of public goods is below Pareto efficiency. These studies endorse the dynamic provision of public goods, but intertemporal consumption/saving choices remain outside of their scope. Tamai (2010) addresses that deficiency by presenting a deterministic dynamic general equilibrium (DGE) model for the provision of public goods. In extending a deterministic model to a DGE model with stochastic disturbances, the supposition of uncertainty and risk is noteworthy. In postulating the many types of risk, it seems natural to consider macroeconomic shocks to productivity, income, and population that are considered in previous empirical studies (Kormendi and Meguire 1985; Grier and Tullock 1989; Ramey and Ramey 1995; Imbs 2007).

This study extends the results of static models concerning the private provision of public goods to include conditions of macroeconomic uncertainty. The main purpose is to investigate the effect of increased risk on the contributions to public goods, including the neutrality result. Furthermore, we analyze the relation between population size and contributions under the negative correlation between population size and volatility in growth reported by Furceri and Karras (2007) and by Alouini and Hubert (2015).

Among its findings are that elevated uncertainty reduces long term contributions to public goods but increases them in the short term. Furthermore, uncertainty related to population size generates a single-peak curve in the relation between population size and contributions by individuals. Contributions by individuals tend toward zero by increasing population in the limit and tend to increase to a finite value by increasing population to a certain level.

2. Model

Consider a closed economy having n persons and identical production technology under stochastic diffusion. Time (t) is continuous. Person i produces final goods over instance (t, dt), dy_i, using private capital. Production technology is common to all individuals and is assumed to be

$$dy_i = Ak_i dt + \sigma Ak_i dz,$$

where dz represents the Wiener process with mean 0 and standard deviation of 1. $A>0$ denotes the expected productivity coefficient and $\sigma > 0$ is the diffusion coefficient. A larger shock affects high-income more than low-income persons. Examples might include windfalls in the stock market or effects of large-scale technological changes in manufacturing or agriculture. Individuals allocate income for private consumption, saving, and contribution to public goods under budget constraint

$$dk_i = [Ak_i - c_i - g_i]dt + \sigma Ak_i dz. \tag{1}$$

The instantaneous utility function is defined over private consumption (c_i) and public goods (G). A public good is producible under linear production technology ($G = \sum_{i=1}^{n} g_i$). The agent chooses c_i and g_i to maximize expected lifetime utility such that

$$E\int_0^\infty U(c_i, G)e^{-\rho t}dt$$

subject to Eq. (1) ($G = \sum_{i=1}^{n} g_i$ and $g_i \geq 0$). We assume the instantaneous utility function is

$$U(c_i, G) = \lambda\frac{c_i^{1-\theta}}{1-\theta} + (1-\lambda)\frac{G^{1-\theta}}{1-\theta},$$

where $0<\lambda<1$, $\theta>0$ and $\theta \neq 1$. When $\theta = 1$, the instantaneous utility function is

$$U(c_i, G) = \lambda \log c_i + (1-\lambda)\log G.$$

We focus on symmetric equilibrium to clarify relations among productivity risk, population size, and contribution to public goods.[5] We obtain this individually optimal rule for private consumption and public goods by solving the maximization problem under symmetric equilibrium (Appendix A)[6]:

$$c = \frac{\lambda^{\frac{1}{\theta}}n}{\lambda^{\frac{1}{\theta}}n + (1-\lambda)^{\frac{1}{\theta}}}(A - \gamma)k, \tag{2a}$$

$$G = \frac{(1-\lambda)^{\frac{1}{\theta}}n}{\lambda^{\frac{1}{\theta}}n + (1-\lambda)^{\frac{1}{\theta}}}(A - \gamma)k, \tag{2b}$$

where

$$\gamma \equiv \frac{A - \rho}{\theta} + \frac{(\theta-1)\sigma^2A^2}{2}. \tag{3}$$

We assume A is sufficiently large to assure positive values for γ and $A > \gamma$. Note that $nk = K \equiv \sum_{i=1}^{n} k_i$ on the right side of Eqs. (2a) and (2b). Considering this equation, Eqs. (2a) and (2b) show that the equilibrium level of private consumption and the provision of public goods are independent of the income distribution. This result is well known as the *Shibata–Warr neutrality theorem* (Shibata 1971; Warr 1983; Bergstrom et al. 1986). We verify this result later.

In contrast, the socially optimal rule for private consumption and public goods under symmetric equilibrium is given as follows (Appendix A):

$$c^* = \frac{\lambda^{\frac{1}{\theta}}n}{\lambda^{\frac{1}{\theta}}n + (1-\lambda)^{\frac{1}{\theta}}n^{\frac{1}{\theta}}}(A - \gamma)k^*. \tag{4a}$$

$$G^* = \frac{(1-\lambda)^{\frac{1}{\theta}}n^{\frac{1+\theta}{\theta}}}{\lambda^{\frac{1}{\theta}}n + (1-\lambda)^{\frac{1}{\theta}}n^{\frac{1}{\theta}}}(A - \gamma)k^*. \tag{4b}$$

Asterisks imply socially optimal values.

Eqs. (1)–(4b) reveal that the equilibrium growth rate of the decentralized economy is identical to that of the centralized economy (Appendix A):

$$\frac{dk}{k} = \frac{dk^*}{k^*} = \gamma dt + \sigma A dz. \tag{5}$$

Eq. (5) implies that saving attains its optimum value. The reason is explained as follows. The contributions to public goods do not affect the production possibility at the current time because public goods only influence the individual's utility. Furthermore, his/her saving does not directly influence others' saving (i.e., accumulation of others' capital) because the rates of return on private capital for others are independent of his/her saving. Then, each individual chooses the saving to

[5] If we assume different degrees of CRRA, the response of G to increased risk differs from that of private consumption. Even though the analysis is complicated in such a case, the relative magnitude between two different degrees of CRRA is important for deriving the result.

[6] If $\sigma = 0$ and $\theta = 1$, Eqs. (2a) and (2b) are reduced to the consumption rules of Tamai (2010) without redistributive taxation. See Turnovsky (2000, Ch.15) and Chang (2004) for the method of solving a stochastic optimization in continuous time.