European Research on Management and Business Economics xxx (2017) xxx–xxx

European Research on Management and Business Economics

www.elsevier.es/ermbe

# An application of extreme value theory in estimating liquidity risk

Sonia Benito Muela [a], Carmen López Martín [b], Raquel Arguedas Sanz [c,*]

[a] National Distance Education University (UNED), Department of Economic Analysis, Faculty of Economics and Management, Senda del Rey, 11, 28040 Madrid, Spain
[b] University Carlos III of Madrid, Department of Statistics, School of Law and Social Sciences, Calle Madrid, 126, 28903 Getafe, Madrid, Spain
[c] National Distance Education University (UNED), Department of Business and Accounting, Faculty of Economics and Management, Senda del Rey, 11, 28040 Madrid, Spain

ARTICLE INFO

*Article history:*
Received 5 October 2016
Received in revised form 21 April 2017
Accepted 2 May 2017
Available online xxx

*JEL classification:*
G32
C14
C22
C53

*Keywords:*
Value-at-risk
Liquidity risk
Extreme value theory
Basel capital accord

ABSTRACT

The last global financial crisis (2007–2008) has highlighted the weaknesses of value at risk (VaR) as a measure of market risk, as this metric by itself does not take liquidity risk into account. To address this problem, the academic literature has proposed incorporating liquidity risk into estimations of market risk by adding the VaR of the spread to the risk price. The parametric model is the standard approach used to estimate liquidity risk. As this approach does not generate reliable VaR estimates, we propose estimating liquidity risk using more sophisticated models based on extreme value theory (EVT). We find that the approach based on conditional extreme value theory outperforms the standard approach in terms of accurate VaR estimates and the market risk capital requirements of the Basel Capital Accord.



## 1. Introduction

As a response to the financial crisis of 2008–2009, the Basel Committee on Bank Supervision (BCBS) proposed a review of the supervisory framework for market risks and introduced new requirements for the trading book (BCBS, 2011a, 2012, 2013, 2016). The new capital requirements assume: (a) a considerable tightening of existing capital requirements; (b) a reduction in arbitrage between bank banking and trading books; and (c) limiting the procyclical effects of such bank capital requirements. The use of internal models was allowed, but the BCBS announced the reconsideration of the VaR concept as the basis of capital requirement for market risk calculations.

The change in the supervisory framework constitutes a response to the fact that during the last crisis, it was found that many financial institutions had insufficient resources to cover the market risk losses they faced during this period. As many such institutions use VaR to quantify their market risk exposure, such results may suggest that this measure may not be appropriate for estimating risk.

Among others factors, the failure of the VaR measure in quantifying risk may be attributable to the fact that many risk management systems estimate VaR from the distribution of portfolio returns computed at the bid-ask average price. This method underestimates risk by neglecting the fact that liquidation occurs not at bid-ask average prices but rather at bid prices. The asset bid price is calculated by adding liquidity costs of an asset to the ask-bid average price. Thus, when liquidity costs are high, which is observed in the financial crisis period, differences between bid and bid-ask average prices become very pronounced. In such cases, estimating VaR using average prices may cause one to underestimate risk.

In taking this into account, the academic literature has proposed incorporating liquidity risk in estimations of market risk (Bangia, Diebold, Schuermann & Stroughair,1999; Ernst, Stange, & Kaserer, 2008, 2009; Stange & Kaserer, 2008). Market liquidity risk emerges as a consequence of changes in liquidity costs. As stated above, these costs can increase dramatically during a financial crisis.

The financial industry and even the BCBS have echoed such proposals (BCBS, 2011b). In this document it is discussed the possibility of requiring financial institutions calculate market risk capital requirement on the bases of VaR adjusted by liquidity risk. In this context, properly measuring liquidity risk is a fundamental task.

In the aforementioned papers, liquidity risk is quantified using the Value at Risk measure. Bangia et al. (1999) defined the liquidity

* Corresponding author.
*E-mail addresses:* soniabm@cee.uned.es (S.B. Muela), carmen.lopez@uc3m.es (C.L. Martín), rarguedas@cee.uned.es (R.A. Sanz).

http://dx.doi.org/10.1016/j.iedeen.2017.05.001

cost as the mid-point of the spread, and they used the VaR of the relative spread as a measure of liquidity risk. Thus, the value at risk adjusted by liquidity costs is calculated by adding the relative spread relative to the price risk.

In this paper, we follow Bangia et al. (1999) by approaching liquidity costs based on the average of the spread, and we estimate liquidity risk as the worst liquidity cost. In their study, Bangia et al. (1999) use a parametric method below a normal distribution to estimate spread VaR. The empirical literature has shown that the spread distribution is far from normal, presenting high levels of skewness and kurtosis. Therefore, the parametric method based on a normal distribution may underestimate liquidity risk. As a way to overcome the drawbacks of this approach, Ernst et al. (2008) propose using a non-normal distribution for relative spread that is estimated from a Cornish–Fisher expansion approximation.

As we show below, the tail of the empirical spread distribution can be adequately characterized by a method based on extreme value theory. Therefore, as a way to estimate properly liquidity risk, we propose using conditional extreme value theory to estimate spread VaR and compare the corresponding results with those obtained by the Ernst et al. (2008) propose. The results indicate that conditional extreme value theory outperforms the parametric method both in terms the accuracy of VaR estimations and of daily capital requirements.

The remainder of the paper is organized as follows. In Section 2, the methodology used to estimate liquidity cost and risk is described. Section 3 presents the empirical analysis conducted. Section 4 present the capital requirements are analyzed. The last section presents our main conclusions.

## 2. Methodology

### 2.1. Liquidity cost

Liquidity in financial markets implies the ability of a particular asset to be traded in the market over a considerably short period of time with a minimal loss of value (Kyle, 1985). Many risk management systems assume that a position can be bought or sold without cost when the liquidation horizon is long enough. However, in real financial markets, liquidity costs can be substantial.

Market risk is basically concerned with describing price/return uncertainty resulting from market movements. Bangia et al. (1999) split risk in the market value of an asset into two components: uncertainty arising from asset returns (pure market risk component) and risk due to liquidity risk.[1]

Liquidity cost is defined as the cost of trading an asset relative to its fair value where the fair value is defined as the bid-ask average price $\left(\mathrm{P}_{\mathrm{mid,t}}\right)$[2]. According to this definition, liquidity cost ($\mathrm{COL}_t$) at time $t$ is calculated as follows:

$$\mathrm{COL}_t = P_t^{bid} - \frac{\left(P_t^{ask} + P_t^{bid}\right)}{2} \tag{1}$$

While taking into account that the bid price is given by ($P_t^{bid} = P_{mid,t} - (1/2)\left(P_t^{ask} - P_t^{bid}\right)$), the liquidity cost (COL) is calculated as follows:

$$\mathrm{COL}_t = -\frac{1}{2}\left(P_t^{ask} - P_t^{bid}\right) \quad \text{or} \quad \mathrm{COL}_t = -\frac{1}{2}P_{mid,t}\frac{\left(P_t^{ask} - P_t^{bid}\right)}{P_{mid,t}} \tag{2}$$

where $\frac{\left(P_t^{ask} - P_t^{bid}\right)}{P_{mid,t}}$ is the relative spread. In the following sections, we denote this expression as $S_t$.

The expression (2) for liquidity cost is correct for small positions but not for larger positions, as market markers are only required to trade positions of up to a certain size at the quoted spread. As a consequence, in the case of larger positions, liquidity cost measured by the average of the spread can be underestimated. In solving this problem, some proposals have been made; see, for instance, Berkowitz (2000), Cosandey (2012) and Giot and Grammig (2006). For a review of these approaches, see Stange and Kaserer (2009). All these proposals consider the fact that liquidity costs increase with the size of the position beyond the quoted spread. The problem with these approaches concerns the data necessary for their implementation, which are not readily available. Spite, the quoted bid-ask spread is not a precise measure of liquidity cost for larger positions, in this paper, we use this approach, as it is overwhelmingly used by companies due to the ease of access to data, thus resulting in cost savings when incorporating liquidity risk while quantifying market risk.

### 2.2. Measuring market risk

Prices and returns are described through the following typical framework:

$$P_{mid,t} = P_{mid,t-1} \times \exp(r_t) \tag{3}$$

where $P_{mid,t}$ is defined as the mid-price at time $t$ and where $r_t$ is the continuous daily mid-price return at time $t$, i.e., $r_t = \ln(P_{mid,t}/P_{mid,t-1})$. In this paper, we use the Value at Risk (VaR) measure to quantify market risk so that we can define the risk price as the relative VaR at the $(1-\alpha)$ percent confidence level over a 1-day horizon:

$$\mathrm{VaR}^{\alpha}_{\mathrm{returns},t} = r_t^{\alpha} = 1 - \exp\left(r_t^{\alpha}\right) \tag{4}$$

where $r_t^{\alpha}$ is the $\alpha$-percentile of daily distribution returns. Thus, $\mathrm{VaR}^{\alpha}_{\mathrm{returns}}$ measures the maximum percentage loss over a 1-day horizon with a confidence $(1-\alpha)$ percent.

In this paper, we use two alternative models to estimate risk price: RiskMetrics (Morgan, 1996), which is a very simple model, and a more sophisticated approach based on conditional extreme value theory (EVT).[3]

Empirical literature show that EVT performs very well in estimating VaR, while RiskMetrics performs very poorly at this task (see Abad, Benito, & López, 2014). In this paper, we use these two models because we wish to evaluate whether the impact of incorporating liquidity risk is dependent on how well we estimate risk prices.

Under RiskMetrics, the Value at Risk of an asset at $\alpha$ % probability can be calculated as:

$$\mathrm{VaR}^{\alpha}_{\mathrm{returns},t} = \mu - k_{\alpha} \times \sigma_t \tag{5}$$

where $\mu$ and $\sigma_t$ are the unconditional mean and conditional standard deviation of the returns; $k_{\alpha}$ is the percentile $\alpha$ of the standard normal distribution. For the estimation of conditional volatility ($\sigma_t$), we use the exponentially weighted moving average model (EWMA) proposed by Morgan (1996). Assuming that financial returns $\{r_t\}$ follow a stochastic process $r_t = \mu + \sigma_t\varepsilon_t$, $\varepsilon_t \sim iii(0,1)$ where $\sigma_t = E\left(\varepsilon_t^2 \middle| \Omega_{t-1}\right)$ and that $\varepsilon_t$ has a conditional distribution function $G(\varepsilon)$ where $G(\varepsilon) = Pr\left(\varepsilon_t < \varepsilon | \Omega_{t-1}\right)$, the Value at Risk of

[1] In this context, liquidity risk is a component of market risk, which is priced in the market (Acharya & Pedersen, 2005).

[2] The mid-price ($\mathrm{P}_{\mathrm{mid,t}}$) is defined as $\frac{\left(\mathrm{P}_t^{\mathrm{ask}} + \mathrm{P}_t^{\mathrm{bid}}\right)}{2}$, with $\mathrm{P}_t^{\mathrm{ask}}$ and $\mathrm{P}_t^{\mathrm{bid}}$ being the best ask-price and bid-price at time $t$, respectively.

[3] EVT is a branch of statistics that addresses extreme deviations from the mean of a probability distribution and limiting probability distributions of such processes. It has been used in fields of engineering, insurance and finance (Embrechts, Küpelberg, & Mikosch, 1999).

Please cite this article in press as: Muela, S. B., et al. An application of extreme value theory in estimating liquidity risk. *European Research on Management and Business Economics* (2017), http://dx.doi.org/10.1016/j.iedeen.2017.05.001