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Related commodity markets and conditional correlations

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Abstract

Related commodity markets have two characteristics: (i) they may be expected to follow similar volatility processes; (ii) such markets are frequently represented by a market aggregate or index. Indices are used to represent the performance and aggregate time series properties of a group of markets. An important issue regarding the time series properties of an index is how the index reflects the corresponding properties of its components, particularly with regard to volatility and risk. This paper investigates the volatility of a market index relative to the volatility of its underlying assets by analysing correlation matrices derived from rolling AR(1)-generalised autoregressive conditional heteroskedasticity (GARCH)(1,1) model estimates. The second moment properties of a linear aggregate of ARMA processes with GARCH errors are analysed and compared with the properties of the individual returns series. Empirical application is made to the markets for non-ferrous metals on the London Metal Exchange (LME). The volatility of the LME Base Metals Index (LMEX) is modelled and compared with the volatility of the 3-month futures contracts for aluminium, copper, lead, nickel, tin, and zinc.

Keywords: GARCH; Volatility; Cross-sectional aggregation; Futures contracts

1. Introduction

Market aggregates (or indices) based on equities, commodities, or other classes of assets are used frequently in finance. Empirical modelling in this paper focuses on the London Metal Exchange Base Metals Index (LMEX) of the London Metal Exchange (LME), an index of spot and futures prices for the six primary metals traded on the LME. The LMEX performs an informational role for participants in the LME

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spot, futures and options markets. Moreover, the index is investible through futures and traded options contracts, based on the level of the index, that are available on the LME. Financial instruments based on commodity market aggregates provide obvious benefits to commodity producers and consumers, who wish to hedge a broad exposure to a particular group of commodities, such as the non-ferrous industrially used metals. In contrast to the futures and options contracts for individual LME metals, those for the index are familiar to financial market participants in that there is no element of physical delivery, and the contracts are cash settled. An index futures contract also provides a convenient vehicle for general investors, such as hedge funds, to gain exposure to industrially used non-ferrous metals markets without having to participate in one of the existing physical, futures or options markets for an individual metal at the LME. Such investors would be interested in the risk of the index relative to the risk in the primary non-ferrous metals markets, that is, the risk relationships between the markets traded on the LME and the index. However, the extent to which returns on an index reflect the volatility properties of returns on the underlying components seems to have received little attention in the literature to date.

In this paper the volatility of the LMEX is compared with that of a subset of its components by analysing the correlations between rolling generalised autoregressive conditional heteroskedasticity (GARCH) processes for each individual series and the index. Estimated parameters, t-ratios, and moment conditions are generated using univariate rolling GARCH models. Correlation matrices are generated for the α and β estimates, their t-ratios, and the second and fourth moment conditions. The estimates, t-ratios and moment conditions of the rolling GARCH model are treated as 'data' in the sense that inferences are drawn regarding the relationships between the index and its components, and between the components themselves, by examining the correlations between the series of estimates, t-ratios, and moment conditions.

2. Time-varying volatility model

Bollerslev's [1] GARCH model is used in the empirical analysis. The GARCH(1,1) specification is the most widely used model in the financial volatility literature, and is considered to represent adequately the observed symmetric intertemporal dependencies in daily returns of many financial time series. The conditional mean of futures price returns is given by the stationary AR(1) model:

$$r_t = \mu + \varphi r_{t-1} + \varepsilon_t, \quad |\varphi| < 1, \tag{1}$$

and the conditional variance of ε_t is given by:

$$\varepsilon_t = \eta_t \sqrt{h_t}, \tag{2}$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{3}$$

where r_t denotes the returns on the futures price from period $t-1$ to t; ε_t the unconditional shock; η_t a sequence of independently and identically distributed random variables with zero mean and unit variance; and h_t the conditional variance of returns. For the GARCH process to exist, $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are sufficient conditions for the conditional variance to be positive. The ARCH coefficient, α, measures short-run persistence in volatility, and the GARCH effect, β, measures the contribution to long-run persistence, namely $\alpha + \beta$.

Several statistical properties have been established for the GARCH(p,q) process in order to define the moments of the unconditional shock. In [5], a necessary and sufficient condition is derived for the existence