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Optimum production and inspection modeling with minimal repair and rework considerations

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ABSTRACT

This paper extends an integrated model of *economic production quantity* (EPQ) and *preventive maintenance* (PM) to incorporate possibilities of minimal repair and rework. Our model determines simultaneously the optimal number of inspections, the inspection interval, the EPQ, and the PM level. Numerical examples of Weibull shock models are given to show that allowing minimal repair and rework will raise the expected profit. Our analyses demonstrate that both minimal repair and rework significantly influence the optimal policy.

1. Introduction

The classical *economic production quantity* (EPQ) model assumes that the production system is free of failures and that all items produced are perfect [1]. Many research efforts have been made to extend the classical EPQ model by loosening various assumptions. In a real world situation, the production process begins by producing lots in an in-control state, and it may become out-of-control by producing defective items. Rosenblatt and Lee [2] considered the effects of process deterioration on the optimal EPQ. They presumed that the time at which the process shifts from the in-control state to the out-of-control state follows an exponential distribution; they also found that the resulting EPQ is smaller than that of the classical model because the use of smaller lots produces fewer defective items. Porteus [3] obtained similar results. Hariga and Ben-Daya [4] extended the model to consider the case in which the deterioration of the process follows a general distribution. Salameh and Jaber [5] presented a modified inventory model which accounted for imperfect quality items when using the EPQ/EOQ formulae. Freimer et al. [6] developed the results that characterize the optimal run length and expected total cost with consideration of reducing setup costs and improving process quality. Jaber [7] also provided a mathematical model with numerical examples to investigate the lot-sizing problem for reduction in setups while considering reworks and interruptions to restore process quality.

Models of production systems must also address the maintenance aspect. Proper *preventive maintenance* (PM) is essential for improving production processes and thus preventing system failure. Most PM models assume that the system is 'as good as new' after each PM action. However, a more realistic approach may model the failure rate of the system after PM as being somewhere between 'as good as new' and 'as bad as old'. Nakagawa [8] and Pham and Wang [9] reviewed the various optimal strategies under imperfect maintenance. Tseng et al. [10] considered a model of imperfect maintenance using a PM that either restores the system to the 'as good as new' state or has the system failing immediately thereafter because of faulty maintenance.

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In general, a system after maintenance is not as good as new, but becomes as if it were younger than its actual age. Rahim [11] explored the optimal control chart design parameter and the EPQ for an imperfect production process. Ben-Daya and Makhdoum [12] discussed the impact of various PM policies when the EPQ model is combined with control chart design.

In a ground-breaking effort, Ben-Daya [13,14] presented an integrated model for the joint determination of EPQ and PM level, thereby capturing the underlying relationship among production maintenance and quality. His model considered the optimal inspection interval, inspection frequency, and production quantity for the general deterioration distribution with a rapidly-increasing failure rate. Sheu and Chen [15] extended Ben-Daya's model to allow for errors in inspections due to human or technological limitations and derived the optimal solution for production quantity and inspection points in relation to inspection errors. Darwish and Ben-Daya [16] studied the effect of inspection errors and PM on a two-stage production inventory system.

Process maintenance is a very important method for quality control in the product manufacturing process. According to past studies, different maintenance strategies have been extended to discuss preventing system failure and improving system reliability. For example, Lee and Rosenblatt [17] pondered the problem of inspection and maintenance strategies of detection delay of dependent repair costs, combining the production plan and maintenance schedule. While supposing that the process reconditioning cost is a function of detection delay and the condition of allowing out-of-control in the production system, and while supposing that the transfer of the process follows exponential distribution, Lin et al. [18] extended their research paper to propose that the distribution of process transfer is an incremental failure rate function. Sheu and Chen [19] extended the research paper of Ben-Daya [14], discussing the optimum production quantity and inspection strategy in the situation of the process allowing for minimal repair. Chen [20] also expanded on the notion of an economic production quantity strategy in which minimal repair and inspection errors exist.

In real-life production systems, the rework option plays an important role in eliminating waste and affecting the cost of manufacturing. For instance, when printed circuit boards are copper-plated, if the thickness of the copper is incorrect, the copper can be washed off and the board replated, which avoids the cost of scrapping the substrate. Hayek and Salameh [21] and Chiu [22] studied the determination of optimal production lot size with reworking of defective items. Flappera and Teunterb [23] showed how reworking plans could both reduce costs and be environmentally friendly. Inderfurth et al. [24] analyzed cost minimizing-scheduling of work and rework processes on a single facility under deterioration of reworkable items. Chen [20] and Chiu et al. [25] proposed a more general model that allowed a certain proportion of reworked units to be scrapped. In this paper, we combine Sheu and Chen's [19] and Chiu's [25] extensions on Ben-Daya [14] to incorporate both minimal repair and rework considerations. Our integrated model determines simultaneously the optimal number of inspections, the duration of the first inspection interval, the EPQ, and the PM level while under the conditions of rework and minimal repair in an imperfect production system. Numerical examples are presented to illustrate important aspects of the proposed model. It is shown that reworking defective items and process minimal repair lead to raising the expected profit.

The rest of the paper is organized as follows. Section 2 presents the model and its mathematical formulation. Section 3 derives the optimal solution. Illustrative examples are given in Section 4 with numerical solutions. Section 5 contains concluding remarks.

2. Model development

2.1. Assumptions

Consider a production process producing a single product. The process is in either the *in-control* state or the *out-of-control* state. At the beginning of a production cycle, the system is assumed to be in an in-control state, producing items of good quality. However, the process may shift to an out-of-control state and be monitored periodically by inspections. The model is developed under the following assumptions:

1. The time which elapses until the production process shifts to the out-of-control state is a random variable and follows a general distribution with increasing hazard rate.
2. The process is inspected at times $t_1, t_2, \ldots, t_k$ to assess its state. If the system is inspected and judged to be in control, PM activities are carried out. The system failure rate will decrease given the PM, and the reduction in the effective age of the system depends on the level of PM performed. The same level of PM is performed during the production cycle and the PM cost is C_{apm}. The PM level, therefore, is a decision variable.
3. On the other hand, if the system is inspected and judged to be out of control, our model allows two types of out-of-control states: (1) For the type I out-of-control state, a minimal repair is performed to restore the system to the in-control state, and the system failure rate remains unchanged. (2) For the type II out-of-control state, a minimal repair is not sufficient—the production has to cease and the system has to be restored to the in-control state and to the 'as good as new' condition by a complete repair or replacement if necessary.
4. The production cycle ends either when the system shifts to a type II out-of-control state or after k inspection intervals whichever occurs first. The number k is a decision variable.
5. When the system is in an out-of-control state, producing items of bad quality, we assume that a portion of the defective items can be reworked and repaired while the rest is regarded as scrap.