Physica A 466 (2017) 502–510

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Determining the multi-scale hedge ratios of stock index futures using the lower partial moments method

Jun Dai [a,*], Haigang Zhou [b], Shaoquan Zhao [c]

[a] School of Management, Wuhan University of Science and Technology, Wuhan 430081, China
[b] Department of Finance, Cleveland State University, Cleveland, USA
[c] Department of International Economics, Business College of Beijing Union University, China

HIGHLIGHTS

- Parametric method with higher moments to compute hedge ratios based on LPM.
- We compare the hedging performance of different methods.
- We study impacting factors for hedge ratios and hedging efficiency, respectively.

ARTICLE INFO

Article history:
Received 10 March 2016
Received in revised form 24 August 2016
Available online 2 October 2016

Keywords:
Downside risk
Wavelet decomposition
Multi-scale hedge ratio

ABSTRACT

This paper considers a multi-scale future hedge strategy that minimizes lower partial moments (LPM). To do this, wavelet analysis is adopted to decompose time series data into different components. Next, different parametric estimation methods with known distributions are applied to calculate the LPM of hedged portfolios, which is the key to determining multi-scale hedge ratios over different time scales. Then these parametric methods are compared with the prevailing nonparametric kernel metric method. Empirical results indicate that in the China Securities Index 300 (CSI 300) index futures and spot markets, hedge ratios and hedge efficiency estimated by the nonparametric kernel metric method are inferior to those estimated by parametric hedging model based on the features of sequence distributions. In addition, if minimum-LPM is selected as a hedge target, the hedging periods, degree of risk aversion, and target returns can affect the multi-scale hedge ratios and hedge efficiency, respectively.

1. Introduction

Rational investors like portfolio managers and some individuals often enter the futures markets with predetermined hedging horizons that vary from seconds to months and beyond. Understanding the relationship between optimal hedge ratios and time horizons is the key to fully exerting the functions of stock index futures. However, previous literature seldom focuses on the effects of different time horizons on optimal hedge ratios. For example, the traditional Ordinary Least Squares (OLS) and recently widely used Bivariate Generalized Autoregressive Conditional Heteroskedasticity (BV-GARCH) models apply both the volatility and the correlation of spot and futures prices in a single period to calculate optimal hedge ratios over different time horizons. Any ignorance of the impact over the hedging horizon on the optimal hedge ratio could be detrimental to decision making and undermine hedging effectiveness.

* Correspondence to: 212 mailbox, 947# Heping Road, Qingshan District, Wuhan University of Science and Technology, Wuhan, Hubei, 430081, China.
E-mail address: daijun78@wust.edu.cn (J. Dai).

http://dx.doi.org/10.1016/j.physa.2016.09.056
0378-4371/

In and Kim [1] were the first to employ the wavelet analysis model to study the relationship between stock and futures markets and to compute hedge ratios over different hedging horizons. Lien and Shrestha [2] decomposed a time series into different scales using the maximal overlap discrete wavelet transform (MOWDT). Multi-scale optimal hedge ratios were then calculated by OLS regressions of the spot wavelet coefficients on the futures wavelet coefficients at different scales. Following the model introduced by Lien and Shrestha [2], Chen et al. [3] examined the performance of multi-scale hedge ratios on the future markets of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX). The empirical results indicated that both hedge ratios and hedge effectiveness went up when hedging horizons increased, and hedge ratios continued to climb until they approached the numerical value of one. This conclusion is consistent with findings in Lien and Shrestha [2]. Conlon and Cotter [4] argued that a utility maximization framework, which incorporates the impacts of degree of risk aversion and hedging horizons on optimal hedge ratios, should be used to determine multi-scale hedge ratios.

In addition, with the growth of investors' risk awareness, drawbacks of variance as a risk measure standard have gradually been perceived because variance, as a traditional risk measure instrument, implies that agents view positive and negative deviation from the expected return as equally undesirable. However, agents frequently perceive risk as a failure to achieve a certain level of return. In this context, downside risk measures, assuming that returns below a certain reference level involve risk and returns above a certain reference level represent better investment opportunities, can be highly relevant. To the best of our knowledge, Sun and He [5] were the first scholars to combine multi-scale hedge ratios with the target of the minimum-LPM hedge. However, their preference to estimate the joint density distribution is a nonparametric method that boasts wide suitability but less efficiency.

In this study, we fill the gap in literature by introducing the parametric estimation method with higher moments including coskewness and cokurtosis to compute multi-scale hedge ratios based on LPM. The new model makes full use of the information from the population like the parametric approach does, reflects the characteristics of non-normal distribution of time series, and allows the estimation of hedge ratios for different horizons.

Our primary empirical results show that, in the CSI 300 index futures and spot markets, the nonparametric universal kernel density method is inferior to parametric methods on the distribution features of time series. If LPM is selected as hedge target, the hedging periods, degree of risk aversion, and target returns can affect the multi-scale hedge ratios and hedge efficiency, respectively.

In the following text, Section 2 constructs the hedging model; Section 3 processes the data from the CSI300 index futures and spot markets; Section 4 expounds upon the empirical analysis and Section 5 draws conclusive findings.

2. Methodology

2.1. A short synopsis of wavelet multi-scale analysis

This section starts from a brief synopsis of the wavelet multi-scale analysis adopted from Percival and Walden [6]. Wavelet analysis, a recently developed signal processing technique in time and frequency domains, has been widely used in the financial engineering fields.

To analyze local features, the given time series should be completely decomposed into different time horizons or frequency components by automatically expanding and contracting the movement of two basic wavelet functions: the father wavelet (or scaling function) $\psi_{j,k}(t)$ and the mother wavelet (or wavelet function) $\phi_{J,k}(t)$, which can be scaled and translated to form a basis for the Hilbert space $L^2(\Re)$ of square integrable functions. The father and mother wavelets are formally defined by the following functions:

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}}\Phi\left(\frac{t-2^j(k-1)}{2^j}\right) \tag{1a}$$

$$\phi_{J,k}(t) = \frac{1}{\sqrt{2^j}}\Phi\left(\frac{t-2^J(k-1)}{2^J}\right) \tag{1b}$$

where $j = 1, 2, \ldots, J$ is the scaling parameter in a J-level decomposition, k is a translation parameter that determines the location of analysis, 2^j is the scale used to evaluate $\Phi(t)$, and t represents time. Given a continuous time series $x(t)$, the wavelet coefficients $d_{j,k}$ and scales coefficients $B_{J,k}$ can be specified by:

$$d_{j,k} = \int x(t)\ \psi_{j,k}\mathrm{d}t, \quad j = 1, 2, \ldots, J \tag{2a}$$

$$B_{J,k} = \int x(t)\ f_{J,k}\mathrm{d}t \tag{2b}$$

where $d_{j,k}$ is the series of detail coefficients obtained from the mother wavelet at all scales from 1 to J, the maximal scale, and $B_{J,k}$ is the series of coefficients from the father wavelet at maximal scale J. For mathematical convenience, the discrete points have the sample size of N, which is assumed to be divisible by 2^J and may be represented by $w = WX$, where W is