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Power available from a depth-averaged simulation of a tidal turbine array

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ABSTRACT

The power available to a tidal stream turbine fence depends on the turbine resistance, the mass flux through the array, as well as the flow speed variation within the array. Depth-averaged simulations are often used to analyse the power available at a site, where the array is modelled as a region of enhanced flow resistance. Without special treatment this method can lead to errors when estimating the power available to the array as the simulated velocity at the turbine plane, and therefore the inferred power, is a spatially averaged value and does not capture flow speed variation within the array; true power being dependent on through-turbine velocity rather than array area-averaged velocity. Linear momentum theory is used to represent the turbines, which by equating the thrust of the numerically simulated array and the analytically modelled turbines enables the true power available to the array to be correctly evaluated. This power is shown to always be less than the erroneous power inferred from the simulated array area-averaged flow velocity. Array end effects, due to the skewed flow passing through the simulated turbine array region, are shown to be particularly important when estimating the power available to short and high-thrust arrays.

The global tidal stream resource has attracted attention in recent years as a potential source of predictable renewable energy, and a large number of turbines deployed in an array or farm configuration will be required to generate a significant amount of power. Recent analytic models and numerical simulations of arrays partially spanning wide channels have shown that there are two scales of flow to be considered for turbine fences: the device-scale flow, the flow around an individual turbine and its wake and remixing with the device scale bypass; and the array-scale flow, the flow around the array and its wake and remixing with the bulk flow through the channel [9,10]. Flow diversion around the array reduces the mass flux through the array which then provides the upstream boundary condition to the device-scale flow problem.

Power is removed from the flow in several stages, and a distinction is made between the power available to a device for conversion into mechanical energy, which is the inviscid limit of the shaft power of a turbine [1], and the power dissipated in mixing processes in the wake. The total power removed from the flow is the sum of the available power and the power dissipated in the wake. Power definitions may be further separated into device and array scales, where the device power is the power available to a single device, and is a function of device-scale thrust and flow speed. The array power comprises the total power available to the tidal devices as well as the power dissipated through device-scale wake mixing. The array power is therefore necessarily greater than the power available to the devices in the array. Estimating the power available to an array thus requires both the device- and array-scale flows to be resolved. The total power removed from the flow is then the sum of the array power and the power lost in array-scale mixing with the channel flow.

The grid resolution in three-dimensional simulations, used to study small groups of tidal devices, ensures that both the devices and their wakes are resolved, allowing the available power to be estimated (see Ref. [8]). However, the computational expense of simulating many tidal devices means that two-dimensional simulations are preferred [2]. The flow is typically described in terms of depth-averaged quantities in these simulations, and representing device-scale flow can be challenging as the grid cells may be larger than the size of a device and/or array [11], making it difficult to determine what local and array flow speeds may be reliably used as reference values.

Relating the power removed from the depth-averaged simulation to the available device power, the maximum power that can be extracted at the turbine plane, is challenging as the relationship

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between the available power and the total power removed from the flow is not straightforward [3]. If turbine design data are available, the relationship can be determined through experiments, numerical simulation, and/or analytic models. Alternatively, an upper bound on the available array power can be established using idealised actuator disks, which act as perfectly efficient momentum extractors; examples include [4] [9], and [11]. The relationship between the array flow speed and disk thrust and power may be determined analytically or through numerical simulations.

This paper develops analytic sub-models for depth-averaged simulations based on linear momentum actuator disk theory (LMADT), which considers momentum and mass conservation in the core and bypass flows past actuator disks, to determine the maximum power available to an actuator disk array. We begin by reviewing challenges in depth-averaged modelling device- and array-scale flows and the different characteristics of the two flows. Analytic corrections are developed based on rigid-lid ([5]) and open channel ([3] and [13]) actuator disk analyses. Drag arising from the support structure is neglected, although this can be modelled as an additional resistance to the flow.

1. Challenges in depth-averaged modelling of tidal turbine arrays

Detailed three-dimensional models are required to resolve the complexities of the flow around a tidal turbine and in its wake. Inevitably, some of these features are lost when models of reduced dimension are used, such as depth-averaged simulations. Thus, the present challenge is how to appropriately account for the unresolved components of the flow and their effect on the thrust and power of the turbine. This paper address three questions arising from this challenge:

1. What is the minimum scale in the flow that can be accurately represented?
2. What is an appropriate reference velocity for parameterising turbine performance?
3. How can momentum be removed from the depth-averaged flow in a manner that is consistent with the underlying three-dimensional case being modelled?

The first question introduces the challenges of developing a numerically stable, grid-resolution independent representation of the turbines, as well as identifying flow phenomenon which cannot be adequately represented in a reduced-order model. The second challenge is to identify an appropriate depth-averaged reference velocity in the numerical simulation, recognising that the flow speed through the turbines may be different to the depth-averaged flow speed at the turbine location, as well as numerical issues with referencing velocities elsewhere in the domain due to parallelisation of the simulation. The third question is closely related to the first two, in which it is necessary to ensure that the simulation is, as much as possible, the depth-averaged equivalent of the three-dimensional process being modelled, by removing the same amount of momentum from the flow. Each challenge is briefly discussed below.

Numerical stability in the turbine simulation region is very important, as erroneous velocities due to numerical instabilities, such as those arising from the magnitude of applied forcing terms and the discretisation method, can result in significant errors in the prediction of turbine thrust and power. As discussed in Section 4, the Discontinuous Galerkin Finite Element Method (DG-FEM) allows step discontinuities in the fluid depth and velocity between adjacent cells, allowing the effects of momentum extraction by the turbines to be determined. Spurious oscillations may develop in the presence of a step discontinuity in a Continuous Galerkin Finite Element (CG-FEM) implementation, requiring a slightly different modelling approach. The numerical representation of the turbines should be grid-independent, insofar as the predicted thrust and power are largely independent of grid size on a converged grid, such as the approach described in Ref. [7].

One challenge in depth-averaged modelling is that the bypass flow around the turbine is not resolved in the vertical dimension, leading to higher depth-averaged velocities at the turbine plane than are seen through the turbine itself, leading to an over-prediction of the thrust and power. It is also challenging to identify an appropriate reference velocity in the domain upstream of the turbines, as the flow speed will be affected by the resistance the turbines apply to the flow, and variations in the domain bathymetry and the influence of boundary conditions makes it difficult to determine an appropriate velocity reference location. Using a non-local velocity may not be computationally desirable if values must be passed between sub-domains in a parallelised model [7]. It is therefore important also to determine a robust and numerically stable method for defining reference flow speeds to determine the turbine performance.

Finally, it is important to correctly represent the three-dimensional turbines in the depth-averaged simulation. The momentum removed from the depth-averaged flow should correspond to the thrust applied to the flow by the array of turbines, a challenge which is related to that of defining an appropriate reference velocity. This requires either a modified bottom drag coefficient (e.g. Ref. [7]), or the application of a depth-averaged force as described below.

Furthermore, it is also important to conserve the blockage ratio, the ratio of turbine frontal area to flow passage cross-sectional area, of the three-dimensional array of turbines in the depth-averaged simulation. Garrett and Cummins [5] showed that the power available to a turbine was, in part, a function of the blockage ratio of the turbine. Similarly, the power available to an array is a function of the blockage ratio of the array in the larger channel. In the case of an axial flow turbine of diameter d, which is necessarily less than the depth of the flow, h, the swept area of turbine, $\frac{1}{4}\pi d^2$, must be represented in a blockage ratio conserving manner in order to ensure correct momentum removal from the flow. In a flow with a prescribed upstream velocity, the available power is over-predicted in the depth-averaged simulation when the blockage ratio is too high, as a greater pressure drop may be achieved in the channel for the same thrust as the blockage is increased. Similarly, if the total energy removed from the channel is prescribed (a change in pressure head is given), then there are fewer mixing losses behind the more highly blocked, depth-averaged turbines for a given thrust, also leading to an over-prediction of the available power. These dynamics are described in further detail in Section 3.

One approach to address these challenges in depth-averaged simulations is to model the turbine array in aggregate, rather than as individual turbines. This approach allows the flow speed through the array to be determined in the numerical simulation, and an actuator disk model is applied, as detailed below, to determine the corresponding flow speed through the turbines that is consistent with the thrust applied to the flow. Furthermore, the actuator disk model ensures the blockage ratio in the depth-averaged simulation is consistent with the three-dimensional array.

2. Separation of scales

Fig. 1 illustrates the separation of array and device scales in a channel with inlet velocity u_∞, where energy extraction can be separated into several stages. The total (global) power removed