Multi-level coarse mesh finite difference acceleration with local two-node nodal expansion method

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ABSTRACT

A computationally efficient and effective multi-level coarse mesh finite difference (CMFD) acceleration is proposed using the local two-node nodal expansion method for solving the three-dimensional multi-group neutron diffusion equation. The multi-level CMFD acceleration method consists of two essential features: (1) a new one-group (1G) CMFD linear system is established by using cross sections, flux and current information from the multi-group (MG) CMFD to accelerate the MG CMFD calculation, (2) an adaptive Wielandt shift method is proposed to accelerate the inverse power iteration of 1G CMFD in order to provide an accurate estimate of the eigenvalue at the beginning of the iteration for both 1G and MG CMFD linear system. Additionally, a nodal discontinuity factor and a diffusion coefficient correction factor are defined to achieve equivalence of the 1G and MG CMFD system. The accuracy and acceleration performance of multi-level CMFD are examined for a variety of well-known multi-group benchmarks problems. The numerical results demonstrate that superior accuracy is achievable and the multi-level CMFD acceleration method is efficient, particularly for the larger, multi-group systems.

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1. Introduction

The computational feasibility of the three-dimensional (3D) whole-core, pin-resolved transport calculation depends on the use of efficient multi-group coarse mesh finite difference (CMFD) method to accelerate convergence of the calculation. This was demonstrated very clearly in the development of the Michigan Parallel Characteristics Transport (MPACT) code (MPACT, 2015) and its application to practical full core Pressurized Water Reactor (PWR) applications. The CMFD solution provided a framework to accelerate the convergence of the eigenvalue problem. In the case of the 2D-1D method used in MPACT, CMFD also provided a natural mechanism to synthesize the 2-D radial MOC transport and the 1-D axial diffusion solution. However, the computational cost of solving the CMFD is still not trivial. In many transport codes that use CMFD procedures (e.g., the MPACT code), obtaining solutions to the CMFD diffusion eigenvalue problem can constitute a significant portion of the computational effort for practical applications (Yee et al., 2017).

The technique of multi-level CMFD acceleration has been implemented in several established core simulators (Downar et al., 2009; Yoon and Joo, 2008; Zhong et al., 2008) and most often a two-group (2G) CMFD has been used to accelerate the multi-group (MG) CMFD calculation. Some research has been performed on the use of a multilevel method with an additional energy grid between the one group and multi groups was investigated in recent research (Cornejo and Anistratov, 2016). However, the work reported here extends previous work by not only implementing a multi-level CMFD method but by also developing an adaptive Wielandt shift (WS) technique in which a 1G CMFD is used to provide an effective estimate of the eigenvalue at the beginning of power iterations for both 1G and MG CMFD linear systems which significantly reduces the number of outer iterations required for convergence. Additionally, the work here introduces two separate factors to achieve equivalence between the 1G and the MG solutions. A nodal discontinuity factor (NDF) and a diffusion coefficient correction factor (DCF), are calculated using the collapsed cross sections, flux and current information from the MG CMFD to achieve consistency of the 1G and MG CMFD solutions. For the MG CMFD linear system, the correction factors NDF and DCF are updated using the local two-node nodal expansion method (NEM). Both the MG and

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1G CMFD linear systems are solved by utilizing an innovative pre-conditioned GMRES solver.

In the following section, the multi-level CMFD acceleration formulation is derived and then in Section 3, a range of well-known multi-group benchmark problems are solved to demonstrate the accuracy and the acceleration performance of the method is assessed.

2. Multi-level CMFD method

2.1. Multi-level CMFD formulation

The 3D multi-group discrete diffusion equation is based on the finite difference method which in the Cartesian coordinate system is given by:

$$\sum_{n \in neighbors} \bar{J}_{g,c,n}^k A_{c,n} + V_c \left(-\sum_{g'} \Sigma_{g,g',c}^k \bar{\phi}_{g',c}^k + \Sigma_{tg,c}^k \bar{\phi}_{g,c}^k - \frac{\chi_g}{k_{eff}} \sum_{g'} v\Sigma_{fg',c}^k \bar{\phi}_{g',c}^k \right) = V_c \bar{S}_{g,c}^{k-1} \tag{1}$$

where $\phi, J, \Sigma, \chi, S, V$ and A are the neutron flux, current, cross section, fission spectrum, source, volume and surface area with standard definitions in nuclear reactor physics. In general, the source is equal to zero for steady state equation. The subscript g, g' are the neutron energy group index and the total number of energy group is G. The superscript k is the number of iteration, and eigenvalue is given as $\lambda = 1/k_{eff}$. The subscripts c and n are the index for the center node and neighbor node, and "c, n" is the index for the interface between node c and node n. The interface current from node c and node n is given by:

$$A_{c,n} J_{g,c,n} = \tilde{D}_{g,c,n} \bar{\phi}_{g,c} - \tilde{D}_{g,n,c} \bar{\phi}_{g,n} \tag{2}$$

where the $\tilde{D}_{g,c,n}$ is used to force the interface current obtained by Eq. (1) to be same as that obtained by a higher order nodal expansion method in the two-node problem. This expression for $\tilde{D}_{g,c,n}$ can be written as:

$$\tilde{D}_{g,c,n} = A_{c,n} \frac{2f_{g,c,n}^{dis}}{\frac{f_{g,c,n}^{dis} h_{c,n}}{f_{g,c,n}^{dif} D_{g,c}} + \frac{f_{g,n,c}^{dis} h_{n,c}}{f_{g,n,c}^{dif} D_{g,n}}}$$

$$\tilde{D}_{g,n,c} = A_{c,n} \frac{2f_{g,n,c}^{dis}}{\frac{f_{g,c,n}^{dis} h_{c,n}}{f_{g,c,n}^{dif} D_{g,c}} + \frac{f_{g,n,c}^{dis} h_{n,c}}{f_{g,n,c}^{dif} D_{g,n}}} \tag{3}$$

An albedo boundary condition is used for the boundary node treatment and $\tilde{D}_{g,c,n}$ is given as the following:

$$\tilde{D}_{g,c,n} = A_{c,n} \frac{2f_{g,c,n}^{dis}}{\frac{f_{g,c,n}^{dis} h_{c,n}}{f_{g,c,n}^{dif} D_{g,c}} + \frac{2}{\alpha}} \tag{4}$$

Here $h_{c,n}$ is the thickness of node c in the direction from node c to n. D_g is the diffusion coefficient. The albedo α has different values for each different boundary condition, e.g. 0.5 for vacuum boundary condition and 0 for reflective boundary condition.

The factors $f_{g,c,n}^{dis}$ and $f_{g,c,n}^{dif}$ are the multi-group nodal discontinuity factor and the diffusion coefficient correction factors, respectively (Xu and Downar, 2012), which can be quantified by using the same expressions as Eqs. (8)–(10) only need to use surface current updated by using local two-node nodal expansion method as described in Section 2.2. The multi-group CMFD linear system can then be written as:

$$V \left(-\sum_{g'} \Sigma_{g,g'}^k \bar{\phi}_{g',c}^k + \tilde{\Sigma}_{tg}^k \bar{\phi}_{g,c}^k \right) + \left(\begin{array}{c} \tilde{D}_{g,c,W}^k + \tilde{D}_{g,c,N}^k + \tilde{D}_{g,c,E}^k \\ + \tilde{D}_{g,c,S}^k + \tilde{D}_{g,c,T}^k + \tilde{D}_{g,c,B}^k \end{array} \right) \bar{\phi}_{g,c}^k$$

$$- \left(\begin{array}{c} \tilde{D}_{g,W,c}^k \bar{\phi}_{g,W}^k + \tilde{D}_{g,N,c}^k \bar{\phi}_{g,N}^k + \tilde{D}_{g,E,c}^k \bar{\phi}_{g,E}^k \\ + \tilde{D}_{g,S,c}^k \bar{\phi}_{g,S}^k + \tilde{D}_{g,T,c}^k \bar{\phi}_{g,T}^k + \tilde{D}_{g,B,c}^k \bar{\phi}_{g,B}^k \end{array} \right)$$

$$= V \left(\bar{S}_{g,c}^{k-1} + \frac{\chi_g}{k_{eff}} \sum_{g'} v\Sigma_{fg'}^{k-1} \bar{\phi}_{g',c}^{k-1} \right) = b_{g,c}^{k-1} \tag{5}$$

where W, E, N, S, T and B represent the west, east, north, south, top and bottom surface of the node. The superscript k is the iteration number. Because of its larger condition number, the MG CMFD linear system converges much more slowly than the 1G CMFD linear system, and therefore it is beneficial to utilize the fission source from the 1G CMFD to accelerate the MG calculation. This will be demonstrated with numerical examples in the following section.

The 1G CMFD linear system can then be derived by collapsing Eq. (5) over all every groups g. At the same time, WS method is used to accelerate outer iteration of 1G CMFD, which then can be written as follows:

$$V \left(\Sigma_a^k \bar{\phi}_c^k - \lambda_s^{k-1} v\Sigma_f^k \bar{\phi}_c^k \right) + \left(\begin{array}{c} \tilde{D}_{c,W}^k + \tilde{D}_{c,N}^k + \tilde{D}_{c,E}^k \\ + \tilde{D}_{c,S}^k + \tilde{D}_{c,T}^k + \tilde{D}_{c,B}^k \end{array} \right) \bar{\phi}_c^k$$

$$- \left(\begin{array}{c} \tilde{D}_{W,c}^k \bar{\phi}_W^k + \tilde{D}_{N,c}^k \bar{\phi}_N^k + \tilde{D}_{E,c}^k \bar{\phi}_E^k \\ + \tilde{D}_{S,c}^k \bar{\phi}_S^k + \tilde{D}_{T,c}^k \bar{\phi}_T^k + \tilde{D}_{B,c}^k \bar{\phi}_B^k \end{array} \right)$$

$$= V \left(\bar{S}_c^{k-1} + \left(\lambda^{k-1} - \lambda_s^{k-1} \right) v\Sigma_f^{k-1} \bar{\phi}_c^{k-1} \right) \tag{6}$$

where λ_s^{k-1} is the shift of $k-1$ inverse power iteration for 1G problem, and except for the special treatment of $\tilde{D}_{c,n}$, all other terms are defined as follows:

$$\bar{\phi} = \sum_g \bar{\phi}_g \tag{7a}$$

$$\bar{J} = \sum_g \bar{J}_g \tag{7b}$$

$$\Sigma_a = \frac{1}{\bar{\phi}} \sum_g \Sigma_{a,g} \bar{\phi}_g \tag{7c}$$

$$v\Sigma_f = \frac{1}{\bar{\phi}} \sum_g \Sigma_{f,g} \bar{\phi}_g \tag{7d}$$

$$D = \frac{1}{\bar{\phi}} \sum_g D_g \bar{\phi}_g \tag{7e}$$

The formula for $\tilde{D}_{c,n}$ is the same as Eqs. (3) and (4). But different from the MG CMFD, the discontinuity factor and diffusion coefficient correction factor for 1G CMFD linear system is determined using interface current from MG CMFD. Once the multi-group current at an interface is known, it is possible to calculate the 1G discontinuity factor and diffusion coefficient correction factor as follows:

$$\Delta \bar{\phi}_{c,n} = \bar{J}_{c,n} h_{c,n} / 2D_c \tag{8a}$$

$$\Delta \bar{\phi}_{n,c} = \bar{J}_{n,c} h_{n,c} / 2D_n \tag{8b}$$

where $\bar{J}_{c,n}$ is one group interface current determined by Eq. (7b) and D is one group diffusion coefficient calculated by using Eq. (7e).

If $\bar{J}_{c,n} \geqslant 0$ and $\bar{\phi}_c - \Delta \bar{\phi}_{c,n} \geqslant \bar{\phi}_n + \Delta \bar{\phi}_{n,c}$, then the one group discontinuity factor $f_{c,n}^{dis}$ and diffusion coefficient correction factor $f_{c,n}^{dif}$ can be calculated as: