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Journal of Economic Dynamics & Control
24 (2000) 855–879

JOURNAL OF
Economic
Dynamics
& Control

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Analysis of global bifurcations in a market share attraction model

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Abstract

In this paper we demonstrate how the global dynamics of an economic model can be analyzed. In particular, as an application, we consider a market share attraction model widely used in the analysis of interbrand competition in marketing theory. We analyze the local and global dynamic properties of the resulting two-dimensional noninvertible dynamical system in discrete time. The main result of this paper is given by the study of some global bifurcations that change the structure of the attractors and their basins. These bifurcations are investigated by the use of critical curves, a powerful tool for the analysis of the global properties of noninvertible two-dimensional maps.

JEL classification: E32; M30

Keywords: Marketing model; Noninvertible maps; Global bifurcations; Critical curves

* Corresponding author. We thank Cars Hommes and two anonymous referees for helpful comments and suggestions. This work has been performed under the auspices of CNR, Italy, and under the activity of the national research project 'Dinamiche non lineari ed applicazioni alle scienze economiche e sociali', MURST, Italy.

0165-1889/00/$ - see front matter
PII: S0165-1889(99)00028-7

1. Introduction

For many years the dynamics of economic systems have been studied by focusing on stable equilibrium behavior. New results in the theory of nonlinear dynamical systems make us aware, however, that fluctuations over time are quite common and may be due to the nonlinear relationships between the variables of the system and not to exogenous stochastic influences. More recently, this viewpoint has been accepted by economists and operations researchers, management scientists and organization theorists alike (see, e.g., Day, 1994; Hommes, 1991; Kopel 1996a; Parker and Stacey, 1994; Thietart and Forgues, 1995).

The question usually addressed in the economics literature is that of the creation of complex attractors through sequences of *local* bifurcations. The study of the *global* bifurcations that cause qualitative changes of the attractors and their basin of attraction has been rather neglected (for recent work on global phenomena in economic models, see Gardini, 1992,1993; Brock and Hommes, 1997; Bischi et al., 1998). Our work moves a step towards this less explored direction. In this paper we show how the global dynamics of an economic model can be analyzed by the study of some global bifurcations that change the shape of the chaotic attractors and the structure of their basins of attraction, as some parameters of the model are varied. These bifurcations are analyzed by the use of critical curves, a powerful method for the investigation of the global properties of noninvertible two-dimensional maps. To have a particular application at hand we consider an attraction model which is widely used in marketing theory, where marketing efforts for the different brands determine their market shares. This economic model is interesting because of the global properties that have not been yet sufficiently explored. It is also particularly apt for our purpose, namely to introduce some concepts for the analysis of two-dimensional discrete dynamical systems by analogies to the well-known one-dimensional quadratic map.

The paper is organized as follows. In Section 2 we present the market share attraction model with two competing brands, and in Section 3 we give some general properties of the resulting two-dimensional noninvertible map. The existence of a unique nontrivial steady state is proved and the creation of complex attractors around the fixed point is numerically evidenced. The concepts of critical curves and basin boundaries are introduced and applied to our model. The main results are given in Section 4, where we analyze some global bifurcations which cause qualitative changes in the structure of the attractors and of their basins as some parameters are allowed to vary. The bifurcations that change the structure of the basins are characterized as contact bifurcations due to tangencies between the critical curves of the noninvertible map and the basins' boundaries, and those changing the structure of the chaotic sets are characterized as homoclinic bifurcations due to a contact between arcs of