Physica A 390 (2011) 3806–3814

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A new approach to quantify power-law cross-correlation and its application to commodity markets

Ling-Yun He*, Shu-Peng Chen

Centre for Futures and Financial Derivatives, College of Economics and Management, China Agricultural University, Beijing 100083, China

ARTICLE INFO

Article history:
Received 23 October 2010
Received in revised form 9 March 2011
Available online 3 July 2011

Keywords:
DMCA
DCCA
Power-law cross-correlation
Crude oil spot markets

ABSTRACT

We proposed a new method: Detrended Moving-average Cross-correlation Analysis (DMCA) to detect the power-law cross-correlation between two correlated non-stationary time series by combining Detrended Cross-Correlation Analysis (DCCA) and Detrended Moving Average (DMA). In order to compare the performance of DMCA and DCCA in the detection of cross-correlation, and to estimate the influence of periodic trend, we generate two cross-correlated time series $x(i)$ and $y(i)$ by a periodic two-component fractionally autoregressive integrated moving average (ARFIMA) process. Then we apply both methods to quantify the cross-correlations of the generated series, whose theoretical values are already known to us. By comparing the results we obtained, we find that the performance of this new approach is comparable to DCCA with less calculating amounts; our method can also reduce the impact of trends; furthermore, DMCA (for background and forward moving average case) outperforms DCCA in more accurate estimation when the analyzed times series are short in length. To provide an example, we also apply this new method to the time series of the real-world data from Brent and WTI crude oil spot markets, to investigate the complex cross-market correlation between these commodity markets. In all, our method is another practical choice to detect the cross-correlation between two short period non-stationary time series, and has potential application to real world problems.

1. Introduction

Empirical evidence supports the existence of fractals or multifractals in commodity or financial markets [1–6]. The existence of multifractality in the markets implies that the scaling geometry of the market patterns may be better described by a spectrum of scaling exponents. Many simultaneously recorded time series in various real world commodity or financial markets are found to exhibit spatial or temporal cross-correlations [3,7–17]. Therefore, in recent years, many researchers attempted to quantify cross-correlations between real systems. Podobnik et al. analyze 1340 members of the NYSE Composite by using random matrix theory, and found the power-law magnitude cross-correlations as a collective mode [16]. Podobnik et al. also find long-range cross-correlations in absolute values of returns between Dow Jones and S&P500 by cross-correlation function [18]. Plerou et al. apply random matrix theory to analyze the cross-correlation matrix of price changes of the largest 1000 US stocks [11]. Richman and Moorman measure the similarity of two distinct physiological time series by means of cross approximation entropy and cross sample entropy [19]. Pincus and Kalman apply approximate entropy to analyze the financial data [8]. However, many previous methods which deal with cross-correlations are mainly based on the dubious assumption that both of the time series are stationary, but now various empirical studies in current

* Corresponding address: Rm. 101, Bld. 3, College of Economics and Management, China Agricultural University, Qinghua Donglu Street, Beijing 100083, China. Tel.: +86 13522821703.

E-mail addresses: lyhe@amss.ac.cn, lyhe@yahoo.cn (L.-Y. He).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.06.013

literature show that many real world time series are non-stationary [6,20,21], which may lead to a spurious detection of auto- or cross-correlation.

Many scientists thereby incorporate the temporal and/or spatial factors and apply Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to investigate the cross-correlation between two non-stationary signals. For example, Jun et al. propose a detrended cross-correlation approach to quantify the correlations between positive and negative fluctuations in a single time series [22]. Based on the previous studies, Podobnik and Stanley propose Detrended Cross-Correlation Analysis (DCCA) to investigate power-law cross-correlations between two simultaneously recorded time series in the presence of non-stationarity [23]. Podobnik et al. then apply their new method to uncover long-range power-law cross-correlations in the random part of the underlying stochastic process [24] and the cross-correlation between volume change and price change [25]. In Ref. [23], DCCA method revealed volatility power-law cross-correlations between absolute values in price changes of Dow Jones and Nasdaq. He and Chen then apply DCCA to investigate nonlinear dependency between characteristic commodity market quantities (variables), especially the relationships between trading volume and market price in many important agricultural commodity futures markets [3]. To unveil multifractal features of two cross-correlated non-stationary signals, Zhou proposes Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to combine Multifractal Detrended Fluctuation Analysis (MF-DFA) and DCCA approaches [26], while the latter can be also regarded as a generalization of Detrended Fluctuation Analysis (DFA). The advantage of DFA lies that it can uncover the scaling features by filtering out polynomial trends. Meanwhile, Detrended Moving Average (DMA), which can filter the trends by the moving average, is based on the Moving Averaging (MA) or mobile average technique proposed by Vandewalle and Ausloos [27,28], and then further developed to quantify long-range auto-correlation in non-stationary fluctuation signals [29–31]. It is well known that the trends may have an influence on the detection of power-law correlation and lead to a crossover [24,32]. Polynomial fitting cannot entirely eliminate the effect of trends in time series. But DMA method can efface the trends by moving average, whose filter is continuous and able to adjust the fitting curve dynamically, by which the estimation accuracy may be increased. In this paper, therefore, we combine the DCCA and DMA, and then propose a new method, Detrended Moving-average Cross-correlation Analysis (DMCA), trying to incorporate the advantages of both DCCA and DMA.

2. Detrended moving-average cross-correlation analysis (DMCA)

Suppose that there are two simultaneously recorded time series $x(i)$ and $y(i)$, $i = 1, 2, \ldots, L$, where L is the length of each time series. We calculate two integrated signals:

$$X(i) = \sum_{k=1}^{i} x(k), Y(i) = \sum_{k=1}^{i} y(k), \quad i = 1, 2, \ldots, L. \tag{1}$$

For a window of size, the moving average is given by:

$$\bar{X}_n(i) = \frac{1}{n} \sum_{k=[-(n-1)\theta]}^{[(n-1)(1-\theta)]} X(i-k), \qquad \bar{Y}_n(i) = \frac{1}{n} \sum_{k=[-(n-1)\theta]}^{[(n-1)(1-\theta)]} Y(i-k) \tag{2}$$

where θ is a position parameter ranging from 0 to 1. In this formulation of moving average Eq. (2), three special cases are considered, namely $\theta = 0$ (backward moving average, in which the filter is obtained by the past data points), $\theta = 0.5$ (centered moving average, in which the filter is obtained by the present data points) and $\theta = 1$ (forward moving average, in which the filter is obtained by the future data points) [31,33]. Then the detrended covariance can be defined as:

$$F^2_{\mathrm{DMCA}}(n) = \frac{1}{L-n+1} \sum_{i=[n-\theta(n-1)]}^{[L-\theta(n-1)]} \left(X(i) - \bar{X}_n(i)\right)\left(Y(i) - \bar{Y}_n(i)\right). \tag{3}$$

If the power-law cross-correlation exists, the following scaling relationship can be observed:

$$F_{\mathrm{DMCA}}(n) \propto n^{H_{\mathrm{DMCA}}}. \tag{4}$$

The Exponent H_{DMCA} can describe the power-law cross-correlation relationship between the two related time series. If $x(i)$ is identical to $y(i)$, this method degenerates into DMA.

As a comparison, let us briefly introduce the algorithm of DCCA [23]:

First, two integrated signals $X(i)$ and $Y(i)$ are calculated by Eq. (1). Then we divide both time series into $L-n$ overlapping boxes, each containing $n+1$ values. For each box that starts at i and ends at $i+n$, the local trends are estimated by linear least-squares fits $\tilde{X}_i(k)$ and $\tilde{Y}_i(k)$. Then, the covariance of residuals can be given by

$$f^2_{DCCA}(n, i) = \frac{1}{n+1} \sum_{k=i}^{i+n} \left(X(k) - \tilde{X}_i(k)\right)\left(Y(k) - \tilde{Y}_i(k)\right). \tag{5}$$