Electric Power Systems Research 143 (2017) 272–280

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Cutting planes based relaxed optimal power flow in active distribution systems

Hongjun Gao[a,b], Junyong Liu[a], Lingfeng Wang[b], Youbo Liu[a,*]

[a] School of Electrical Engineering and Information, Sichuan University, 610065 Chengdu, China
[b] Department of Electrical Engineering and Computer Science, University of Wisconsin–Milwaukee, 53211 Milwaukee, WI, USA

ARTICLE INFO

Article history:
Received 7 August 2016
Received in revised form 8 October 2016
Accepted 14 October 2016

Keywords:
Active distribution system (ADS)
Second-order cone programming (SOCP)
Branch flow model (BFM)
Cutting planes
Relaxation exactness

ABSTRACT

Optimal power flow (OPF) has played a significant role in the design, planning and operation of active distribution systems (ADSs). Due to its ability to achieve the optimality with higher computational efficiency, the second order cone programming (SOCP) based on branch flow model (BFM) has received an increasing attention in recent years. However, various sufficient conditions and assumptions are required to ensure the relaxation exactness in the existing literatures. In this study, we introduce the cutting planes to tackle this exactness challenge for general distribution networks especially the ADS with high renewable penetration. Firstly, a typical operation optimization model is presented as an example of OPF in ADS. A general branch flow model based relaxed optimal power flow (BFM-ROPF) model is then formulated as a SOCP problem after conic relaxation. According to these conditions with the objectives which are not monotonously increasing in power injections or branch currents, a total power loss cut (TPLC) is introduced to ensure the conic relaxation exactness. Moreover, a leaf branch current cut (LBCC) is incorporated to prevent the inexactness of SOCR in some leaf branches. Afterwards, the proof of the cutting planes is given to guarantee the optimality and the relaxation exactness of the BFM-ROPF model even though power loss is not included in the objectives. Numerical results based on 33-bus and 69-bus systems are given to verify the effectiveness of the proposed method.

1. Introduction

With the higher penetration of distributed energy resource (DER) into the distribution system, the current passive distribution networks are being transformed into active distribution systems (ADS). As a result, the design, planning and operation of the contemporary distribution system are becoming much more complicated than the traditional one as various active devices should be scheduled simultaneously. Generally, optimal power flow (OPF) is a fundamental approach required to support decision-making by performing optimization for several objectives under certain physical and operating constraints [1,2]. Also, the initiative for promoting ADS demands more effective OPF methods with higher computing efficiency and higher solution accuracy.

In Ref. [3], the AC optimal power flow (AC-OPF) is emphasized due to the phenomena related to voltage magnitudes and reactive power which are of great relevance to distribution systems. Besides, in the distribution network some reasonable simplifications such as the linearization of power flow constraints which are commonly used in transmission systems are not applicable any more. Therefore, in order to address the challenges that the non-linear power flow equation constraints bring to distribution engineers, a considerable number of studies have been carried out on this topic which could be generally divided into two categories: heuristic algorithms and analytical methods [4,5]. The heuristic algorithms have been paid much attention due to their ease of implementation, which mainly include genetic algorithm [6], Tabu search [7], particle swarm optimization method [8], etc. But these heuristic algorithms have a few major disadvantages (e.g., high computing cost), which largely reduce their adaptability and effectiveness in handling problems of ADS equipped with massive DERs. Therefore, more studies are focused on exploring the analytical methods in these decades [9], which have been developed primarily to solve the OPF in transmission networks. Traditional analytical methods for solving the highly non-linear problem include quadratic programming [10], Newton Raphson [11], interior point method [12], Lagrangian Relaxation [13] etc. Compared to those traditional analytical methods, convex programming becomes widely

---

* Corresponding author at: School of Electrical Engineering and Information, Sichuan University, Chengdu 610065, China.
E-mail addresses: gaogavin1989@gmail.com (H. Gao), liuyoubo@scu.edu.cn (Y. Liu).

http://dx.doi.org/10.1016/j.epsr.2016.10.040
0378-7796/© 2016 Elsevier B.V. All rights reserved.

Nomenclature

Indices	
i, j, h	Index of buses
ij	Index of branches
k	Index of iteration
Sets	
B/B^{Load}	Set of system buses/load buses
$B^{\mathrm{ESS}}B^{\mathrm{WTG}}/B^{\mathrm{SVC}}/B^{\mathrm{TR}}$	Set of buses connected with ESS/WTG/SVC/transformer
$\delta(j)$	Set of buses whose parent is bus j
E	Set of branches
$\mathrm{P}_i/\mathrm{Q}_i$	Set of active/reactive power injections
Z	The feasible set to the original model
Z_k	The feasible set in the k-th iteration
Parameters	
r_{ij}/x_{ij}	Resistance/reactance of branch ij
$c^{\mathrm{Loss}}, c_t^{\mathrm{TR}}, c^{\mathrm{WTG}}$	Price to power loss, main grid power and wind power curtailed
$C^{\mathrm{Loss}}, C^{\mathrm{TR}}, C^{\mathrm{WTG}}, C$	Cost due to power loss, main grid power and wind power curtailed, and total cost
$\underline{P}_j^{\mathrm{TR}}/\bar{P}_j^{\mathrm{TR}}$	Lower/upper bound of main-grid active power at bus j
$\underline{Q}_j^{\mathrm{TR}}/\bar{Q}_j^{\mathrm{TR}}$	Lower/upper bound of main-grid reactive power at bus j
$\underline{P}_j^{\mathrm{ESS}}\ \bar{P}_j^{\mathrm{ESS}}$	Lower/upper bound of power limit of ESS connected to bus j
$\bar{I}_{ij}$	Current capacity limit of branch ij
$\underline{V}_j/\bar{V}_j$	Lower/upper bound of voltage magnitude at bus j
$\underline{Q}_j^{\mathrm{SVC}}/\bar{Q}_j^{\mathrm{SVC}}$	Lower/upper bound of reactive power for SVC connected to bus j
$P_j^{\mathrm{WTG,FRE}}$	Forecast output of wind power connected to bus j
$P_j^{\mathrm{Load}}/Q_j^{\mathrm{Load}}$	Active/reactive load for bus j
$\bar{l}_{ij}/\bar{v}_j/\underline{v}_j$	The bound of branch current and bus voltage
Variables	
$P_j^{\mathrm{TR}}, P_j^{\mathrm{ESS}}$	Active power for transformer bus, ESS bus j
$\Delta P_j^{\mathrm{WTG}}$	Wind power curtailed at for bus j
$Q_j^{\mathrm{TR}}, Q_j^{\mathrm{SVC}}$	Reactive power for transformer bus, CB bus, SVC bus j
P_{ij}/Q_{ij}	Active/reactive power flow from bus i to bus j
I_{ij}/V_j	Current magnitude of branch ij and voltage magnitude of bus j
l_{ij}/v_j	Square of current magnitude of branch ij and square of voltage magnitude of bus j
$\boldsymbol{p}/\boldsymbol{q}$	Vector of the active/reactive power injections
$\boldsymbol{P}/\boldsymbol{Q}$	Vector of the active/reactive powers in the branches
$\boldsymbol{l}/\boldsymbol{v}$	Vector of the branch currents and bus voltages
$\dot{\boldsymbol{x}}$	The optimal solution of original model
$\boldsymbol{x}_k$	The solution in the k-th iteration

used because it is able to achieve the optimal solution in polynomial time by the means of the convexification of power flow equations [14–16]. The semidefinite programming (SDP) was first introduced in [17] to solve the OPF for mesh networks based on the bus injections model. Compared to SDP, the second-order cone programming (SOCP) has attracted more attention for radial distribution networks because of its much lower computational complexity [18,19]. A SOCP based load flow model is first proposed in [20], where the convex optimization problem is solved by interior point methods. Moreover, another type of the SOC relaxation (SOCR) approach in radial distribution systems is investigated in Refs. [21,22], which is based on the branch flow model (BFM). However, it is just the relaxation process that may result in inexact solutions in this branch flow model based relaxed OPF (BFM-ROPF) [23,24]. The optimization result is meaningless when the SOCR is not exact. Some sufficient conditions are given in Refs. [25,26] for radial networks (e.g., not both constraints on active and reactive power injections are binding at both ends of a line).

For the active distribution system with high penetration of DERs, the effectiveness of the SOCR is also verified in Ref. [27] using four IEEE benchmark systems and realistic systems. But only under a few specific strong conditions, the relaxation is exact (e.g., the objective function is convex and monotonically increasing in each active power injection and the initial OPF is feasible [28]). Moreover, in order to accommodate more renewable energies [29,30], some other objectives (e.g., renewable energy curtailment) should be taken into account in ADS operations, which may lead to the inexactness of SOCR because the objective function is not monotonically increasing in power injections of these buses with renewable energies. Meanwhile, it is common to include the penalty cost of renewable energy curtailed in objective functions to represent the interests of DG operators in a number of ADS studies.

As mentioned earlier, though many restrictive assumptions are given to guarantee the exactness of SOCR, no recourse is provided to obtain a feasible solution if these conditions are not satisfied or the computing results are not exact. A spatial branch and bound algorithm is proposed in Ref. [31] to achieve the optimality of SDP in several standard IEEE systems without any premises. However, it may be fairly difficult to compute the upper bound of the original nonlinear problem in the practical distribution networks by using CONOPT, which is a nonlinear programming (NLP) solver. Another trial implementation is an increasingly tight linear cut applied to the relaxed model. The cutting planes are previously applied to solve the relaxation problems for integer programming. In each iteration, a set of linear cuts is added until the convergence tolerance criterion is fulfilled. The cuts can eliminate the solution returned in the previous iteration from the feasible set when the relaxation is not exact. The cutting planes are first studied in Ref. [28] to ensure the exactness of relaxed optimal power flow in the radial distribution system. The relaxation gap may shrink to meet the desired requirement after several iterations. Yet, in Ref. [28] the study is conducted based on the premise that the relaxation is exact when the power loss is the single objective. Actually, in the practical test there may exist some SOCR gaps at some leaf branches even though the power loss is the single objective in the OPF model. In this work, we focus on obtaining the optimal solution in a given distribution system without any restrictive assumptions or sufficient conditions based on BFM-ROPF. First, a typical optimization model by BFM is built as an OPF example in ADS, where the high penetration of renewable energies is especially considered. The SOCP problem is then formulated for the general BFM-based OPF model after SOCR. In addition, in order to meet the exactness requirement, cutting planes including the total power loss cut (TPLC) and leaf branch current cut (LBCC) are applied to the relaxed OPF model in the iteration process. Then, the validity of cutting planes is proved to ensure the SOCR exactness for general situations. Afterwards, the applicability of the proposed cutting planes for relaxation exactness is investigated through carrying out numerical case studies. Specifically, the main contributions of this paper are threefold. First, a typical operation optimization model is used as an example in ADS with high penetration of renewable energies and active network management devices. Then the general relaxed optimal power flow model based on BFM is generated after the conic relaxation. Second, two types of cutting planes including TPLC and LBCC are introduced to ensure the exactness of the relaxed OPF model in the general con-