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Collective optimization problems with optimal decentralized selfish strategies ⋆

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Abstract: In this paper, we consider cooperative multi-agent systems minimizing a social cost. Cooperation is induced by the cost that penalizes the divergence of each agent from the average collective behavior. In principle, the optimal solution is centralized and requires a complete communication graph. We study this problem for two important system input-output norms, the ℓ_1 induced norm and the per-agent $\mathcal{H}_2$ norm squared, as function of the number of agents, n, and various cost structures. For the case of identical agents, and the simplest cost setting, we show that the optimal social solution is always the optimal decentralized selfish solution. For more general cost functions, we show that the optimal solution is always decentralized in the ℓ_1 induced norm case. In the case of the per-agent $\mathcal{H}_2$ norm squared cost, we show that the optimal decentralized selfish solution is socially optimal in the limit of large n. All of these also hold for several classes of problems with nonidentical agents. In simple terms, these results, identify important problem classes where decentralized/selfish behaviors are socially optimal, and for which inter-agent communication is or becomes unnecessary for large n.

Keywords: Distributed control, multi-agent systems, cooperative optimization.

1. INTRODUCTION

The study of networked systems has become a main area of research in recent years. There is a plethora of open questions pertaining to various aspects and properties of systems defined on graphs. These include, the characterization of system's structural properties like controllability/observability Liu et al. (2011), Rahnami et al. (2009) and Pasqualetti et al. (2014), performance, or noise and uncertainty amplification Bamieh et al. (2012), Wang and Elia (2012), Siami and Motee (2013), distributed controller design Voulgaris (2001); Rotkowitz and Lall (2006); Langbort et al. (2004); Vamsi and Elia (2016) among others. In this paper, we are interested in originally disconnected multi-agent systems that share a common social objective. It is the social objective that induces or requires a network structure to exchange information and make optimal decisions. In particular, we consider cost functions with two specific characteristics: the first is that they represent input-output performance measures of the overall system; the second characteristic is that the cost functions are function of the distance of each agent's output from the corresponding average output of the collective. This second characteristic is related to that of cost functions used in Mean Field games e.g., Huang et al. (2007); Huang et al (2012); Lasry et al. (2007); Nourian et al. (2010); Moon and Basar (2014); Bauso et al. (2016); Wang et al. (2014).

In general, the optimal controller is network distributed and requires each agent to exchange information with all the others agents. For this reason, such optimal controller is called centralized in some papers. However, this solution does not scale well when the number of agents, n, is large.

Two questions become relevant in this settings:

1) When is the optimal solution actually decentralized? In particular, social cost functions that are optimized by selfish agents are of interest in various fields, as they prove that selfish behavior is socially optimal. Moreover they do not require any underlying communication network and the cost associated with it.
2) When are localized or even decentralized solutions a good approximation of the optimal centralized one?

With respect to the second question, we point out the approximation method used in the $\mathcal{H}_2$ norm minimization of spatially invariant systems Bamieh et al (2002). Therein it is shown that the optimal controller has exponentially decaying tails in the spatial domain. The control strategy can then be approximated by a local one by truncating the tails. In the mean field approaches, Huang et al (2012), a decentralized suboptimal strategy is obtained by replacing the actual average measurements with a deterministic input representing the mass behavior. This input under appropriate assumptions is shown to well approximate, in the limit of large n, the expected value of the average

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measurement signals, and can be computed independently by each agent.

In this paper, we focus on the first question. We are interested in identifying how decentralization, and hence lack of communication, between a large number of dynamically decoupled LTI agents affects the overall closed loop performance, captured by input-output norms that encode deviation from some average type of behavior. We take an input-output approach and obtain results in various norm settings and types of population. In particular, we consider the ℓ_1-induced norm and the per agent $\mathcal{H}_2$ norm squared.

In the case of identical agents, we show that the optimal solution is always decentralized for any n. Thus, complete decentralization does not degrade the optimal performance. This is also the case for nonidentical agents when these different agents have the same inner factors, e.g., same unstable poles and zeros with SISO dynamics. In the more general case of nonidentical agents and when ℓ_1-induced norm is used, it is also shown that the loss due to decentralization becomes marginal as the agent population size grows.

The per-agent $\mathcal{H}_2$ norm square of the overall system measures the power contribution of each agent to the total cost when the inputs are time and space uncorrelated with bounded covariance. For identical agents, we similarly show that loss due to decentralization becomes marginal for large n. Within the body of our analysis for the identical agents case, we also considered a relevant stochastic, state space $\mathcal{H}_2$ problem formulation; its solution provides insight to the way that the loss, due to decentralization, of the per agent normalized performance becomes marginal for large numbers of agents.

The paper is organized as follows. In section 2, a basic norm minimization problem is posed in terms of a Youla-Kucera (Y-K) parametrization of all stabilizing, possibly centralized, controllers. In section 3, we proceed to its solution as well as to the solutions of more complicated versions for the case of identical agents. In section 4, we analyze the case of nonidentical agents and conclude in section 5.

Some basic notation is as follows. For a real sequence $M = \{M(k)\}_{k=0}^{\infty}$ we use the ℓ_1-norm $||M|| := \sum_k |M(k)|$ and the ℓ_2 or $\mathcal{H}_2$ norm $||M||_2 := [\sum_k M(k)^2]^{1/2}$. For a real sequence of matrices $M = [M_{ij}] = \{M(k)\}_{k=0}^{\infty}$ we use the ℓ_1-induced norm $||M|| := \max_j \sum_i ||M_{ij}||$ and the ℓ_2 or $\mathcal{H}_2$ norm $||M||_2 := [\sum_{i,j} ||M_{ij}||_2^2]^{1/2}$. These norms will be used for norms of LTI systems when viewed in terms of their pulse response. Also, in the discussions that follow we often use the $|| \bullet ||$ notation generically up to a point before we clearly specify what are the norms that the results apply.

2. PROBLEM DEFINITION

We consider n dynamically decoupled systems $\{G_i\}_{i=1}^{n}$. Each G_i has control input u_i, measurement output y_i, disturbance w_i and regulated variable z_i. Let $z = \Phi w$ where $z = [z_i]_{1\le i\le n}$, $w = [w_i]_{1\le i\le n}$ are respectively the vectors of regulated and disturbance signals, and Φ is the closed loop when each G_i is in feedback with its corresponding controller K_i. We allow at this point each K_i to be connected to the other controllers K_j, thus the overall controller K, given by the relation $u = Ky$ where y and u are the concatenated measurements and control signals y_i and u_i respectively, can be a full matrix. For any K that stabilizes the overall system of G_i's the corresponding Φ can be obtained via a Youla-Kucera parametrization as

$$\Phi = w \mapsto z = H - UQV$$

where $H = \mathrm{diag}(H_1, \dots, H_n)$, $U = \mathrm{diag}(U_1, \dots, U_n)$, $V = \mathrm{diag}(V_1, \dots, V_n)$ are diagonal stable systems the elements of which can be obtained from standard factorizations of the individual G_i's. The system Q is also stable but can be a full matrix of stable systems $[Q_{ij}]_{1\le i,j\le n}$. In view of this we have

$$\Phi = \begin{bmatrix} H_1 - U_1Q_{11}V_1 & -U_1Q_{12}V_2 & \cdots \\ -U_2Q_{21}V_1 & H_2 - U_2Q_{22}V_2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{1}$$

We are interested in optimizing system's performance with respect to a variable that measures deviations from population average. In this sense, defining

$$e_i := z_i - \bar{z}, \quad \bar{z} := \frac{1}{n}(z_1 + \cdots + z_n), \quad e := [e_i]_{1\le i\le n}$$

we are interested in the map

$$\Psi := w \mapsto e$$

which can be expressed as

$$\Psi = \Phi - \bar{\Phi}$$

with

$$\bar{\Phi} = \frac{1}{n}\mathbf{1}\mathbf{1}^T\Phi = [\mathbf{1}\bar{\Phi}_1 \dots \mathbf{1}\bar{\Phi}_n]$$

where $\mathbf{1}$ is a vector of 1's [1] and

$$\bar{\Phi}_j = \frac{1}{n}H_j - \frac{1}{n}(U_1Q_{1j}V_j + U_2Q_{2j}V_j + \cdots + U_nQ_{nj}V_j)$$

A basic problem of interest is to find the controller to minimize some norm of Ψ i.e.,

$$\psi^o := \inf_Q ||\Psi|| \tag{2}$$

In later sections we consider more elaborate problems which involve norms of other than deviation form average signals.

3. IDENTICAL SYSTEMS

We first consider the case where the systems G_i are identical in which case $H_i = \bar{H}$, $U_i = \bar{U}$, and $V_i = \bar{V}$ for all $i = 1, \dots n$. Looking at a fixed j,

[1] or, a n-block vector of identity matrices I of appropriate dimension in the case of vector outputs z_i