Journal of Petroleum Science and Engineering 162 (2018) 150–157

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Reliability analysis models for hydraulic fracturing

Hongbo Zhao [a,*], Zhen Li [a], Changxing Zhu [a], Zhongliang Ru [b]

[a] School of Civil Engineering, Henan Polytechnic University, Jiaozuo, 454003, People's Republic of China
[b] College of Resources and Material Sciences, Taiyuan University of Science and Technology, Taiyuan, 030024, People's Republic of China

ARTICLE INFO

Keywords:
Hydraulic fracturing
Uncertainty
Reliability analysis
First order reliability method

ABSTRACT

For many decades, hydraulic fracturing has been the technique most commonly used in the oil and gas industry to increase the productivity of low permeability reservoirs. However, much uncertainty remains when applying this technique, which can influence performance. It has proved difficult, for example, to evaluate the uncertainties inherent in traditional deterministic hydraulic fracturing models using classic analytical and numerical methods. Thus, in this study, we adapted reliability analysis to evaluate the performance of hydraulic fracturing including the length, width, and nature of a fracture when considering uncertainty. Random variables are assumed to follow a normal distribution, and the first-order reliability method (FORM) was used to calculate a reliability index for hydraulic fracturing based on Khristianovic-Geertsma-de Klerk (KGD) and Perkins-Kern-Nordgren (PKN) models. Reliability indexes obtained under different conditions show excellent agreement with Monte Carlo simulations, and indicate that use of this method provides an efficient and effective scientific approach for the uncertainty analysis of hydraulic fracturing. We also investigate and discuss the influence of elastic mechanical parameters of rock formations as well as *in situ* stress as random variables affecting the performance of hydraulic fracturing alongside various other factors that can influence this process, including rock tensile strength, injection rate and time. This analysis provides quantitative insights into the uncertainties that underlie hydraulic fracturing models and enables the identification of potentially influencing factors to reduce performance uncertainties. The results also show reliability analysis provides an appropriate solution for evaluation of uncertainty and can further improve the performance of hydraulic fracturing.

1. Introduction

Hydraulic fracturing has been commonly applied for decades in the oil and gas industry to increase the productivity of low permeability reservoirs. This method can broadly be defined as the process by which a fracture initiates and propagates as the result of hydraulic loading (i.e., pressure) applied via a fluid (Adachi et al., 2007). Hydraulic fracturing has also been applied in other geomechanical fields, including for the disposal of waste drill cuttings underground (Moschovidis et al., 2000), heat production from geothermal reservoirs (Harlow and Pracht, 1972), CO_2 sequestration (Boschi et al., 2009), coal bed methane recovery (Heo et al., 2015), gas control in coal mines (Li et al., 2015), and for the *in-situ* characterization of stress (Desroches, 1995).

This process, however, represents a multi-scale and multi-physical problem. Hydraulic fracturing of reservoir media under transient dynamic loadings comprises coupled mechanical and thermal processes as well as fracturing and filtration. Of these, mechanical processes include irreversible deformation, fluid-rock interactions, and the flow of fluids with phase transitions, while the process of fracturing involves the formation, movement, interaction, and accumulation of micro-structural damage (i.e., pores, cracks). However, these processes and the way they interact are very complicated and uncertain; while the dimensions and propagation characteristics of a hydraulic fracture are crucial to the design of operations, these parameters are very difficult to determine in practice. Accurately determining the shape and dimensions of propagating fractures are critical challenges in hydraulic fracturing.

A number of simplified analytical hydraulic fracturing models have been developed in recent decades, including two classical constant height models, the Khristianovic-Geertsma-de Klerk (KGD) model (Khristianovic and Zheltov, 1955; Geertsma and Klerk, 1969) and the Perkins-Kern-Nordgren (PKN) model (Perkins and Kern, 1961; Nordgren, 1972), in addition to pseudo three-dimensional (3-D) models (Settari and Cleary, 1986). These models, however, typically make very simple assumptions about crack geometry, the criteria governing crack origination, and the dynamics of crack propagation, internal fluid flow, formation, and fluid properties. In reality, due to the complexity of

* Corresponding author.
E-mail address: bxhbzhao@hotmail.com (H. Zhao).

https://doi.org/10.1016/j.petrol.2017.12.048
Received 23 February 2017; Received in revised form 1 December 2017; Accepted 14 December 2017
Available online 15 December 2017

Nomenclature

β reliability index σ_{hmin} least horizontal principal stress (MPa)
X set of random variables σ_{Hmax} greatest horizontal principle stress (MPa)
x_i random variables P_b breakdown pressure (MPa)
μ mean values T rock tensile strength (MPa)
C covariance matrix P_p pore pressure (MPa)
F failure domain α Biot poroelastic parameters
R correlation matrix υ Poisson's ratio
σ_i standard deviation L_l limiting values of fracture length (m)
μ_i^N equivalent normal mean w_l limiting values of maximum width (m)
σ_i^N equivalent standard deviation L fracture length (m)
$F(x)$ original non-normal CDF w maximum induced width (m)
φ^{-1} probabilistic density function p_w pressure of wellbore (MPa)
$f(x)$ original non-normal probability density Q injection rate (m^3/s)
$\phi()$ cumulative distribution function μ fluid viscosity(Pa·s)
c_l fluid loss coefficient h fracture height (m)
n_i varying dimensionless number t time (s)
G shear modulus of the rock (MPa) Φ^{-1} inverse of standard normal cumulative distribution

hydraulic fracturing, this process has proved difficult to characterize using analytical models. A number of additional numerical methods, including the use of finite (FE) and boundary elements, have been proposed as useful tools to model the propagation of hydraulic fractures (Papanastasiou, 1999; Garagash, 2006; Lecampion and Detournay, 2007; Zhang et al., 2011; Carrier and Granet, 2012; Hunsweck et al., 2013); available numerical approaches were reviewed by Adachi et al. (2007). Recently, an extended FE (XFEM) and discrete element methods have also been applied to simulate hydraulic fracturing (Choi, 2003; Lecampion, 2009; Gordeliy and Peirce, 2013; Deng et al., 2014; Wang, 2015).

Presently available works that deals with the origination and propagation of cracks in high velocity fluid flow regimes are very limited in terms of physical and geometrical descriptions of phenomena, mechanisms, uncertainties, and optimization analysis (Lukyanov and Chugunov, 2014). The limitations of available analytical and numerical methods lie in the fact that they do not take into account uncertainties in variables such as parameters of rock mechanics, *in situ* stress, and injection rate. Thus, to evaluate the uncertainties that underlie hydraulic fracturing, Lukyanov and Chugunov (2014) applied global sensitivity analysis to quantify and rank uncertainties in the performance metrics of predicted fracturing processes, building on earlier work by Rutqvista et al. (2000), who analyzed the uncertainties involved in the determination of *in situ* stress by hydraulic fracturing. Previous work has shown that it is essential to take into account uncertainties in hydraulic fracturing because these exert considerable influence on the shape and dimensions of propagating fractures.

Reliability analysis is one method that can be used to appropriately evaluate uncertainty. This analytical approach was developed and has been successfully applied in rock mechanics and engineering (Hoek, 1998; Oreste, 2005; Mollon et al., 2009; Li and Low, 2010; Lv and Low, 2011; Zhao et al., 2014). We adapted the reliability analysis method in this study in order to characterize the uncertainties involved in hydraulic fracturing, applying first-order reliability methods (FORM) to quantify uncertainties. We calculated and analyzed values for the reliability index as well as probabilities of failure to provide quantitative insights into the uncertainties underlying models for hydraulic fracturing and to identify factors that contribute to reductions in the performance of hydraulic fracturing.

2. First order reliability method that apply varying dimensionless numbers

The Hasofer-Lind index (β), is widely used in reliability analysis (Hasofer and Lind, 1974). The matrix formulation for a correlated normal of this index is calculated as follows:

$$\beta = \min_{X\in F}\sqrt{(X-\mu)^T C^{-1}(X-\mu)}, \tag{1}$$

In this expression, X is the vector representing the set of random variables x_i, while μ is the vector of mean values, C is the covariance matrix, and F is the failure domain. Eq. (1) provides the minimum distance in units of directional standard deviation from the mean-value point of random variables to the boundary of the limit state surface.

Low and Tang (1997a, 1997b) presented an alternative interpretation of β based on the perspective of an expanding ellipsoid in the original space of the basic random variables, expressed as follows:

$$\beta = \min_{X\in F}\sqrt{\left[\frac{x_i-\mu_i}{\sigma_i}\right]^T [R]^{-1}\left[\frac{x_i-\mu_i}{\sigma_i}\right]}, \tag{2}$$

In this expression, $[R]$ is the correlation matrix, and σ_i denotes the standard deviation of random variable x_i.

In the case of correlated non-normals, an ellipsoidal perspective remains valid if Eq. (2) is rewritten as follows:

$$\beta = \min_{X\in F}\sqrt{\left[\frac{x_i-\mu_i^N}{\sigma_i^N}\right]^T [R]^{-1}\left[\frac{x_i-\mu_i^N}{\sigma_i^N}\right]} \tag{3}$$

In this expression, μ_i^N and σ_i^N denote the equivalent normal mean and equivalent normal standard deviation of random variable x_i, respectively. Values for ${\mu^N}_i$ and ${\sigma^N}_i$ can be computed by applying the Rackwitz-Fiessler two-parameter equivalent normal transformation (Rackwitz and Fiessler, 1978), as follows:

$$\sigma^N = \frac{\varphi\{\Phi^{-1}[F(x)]\}}{f(x)} \tag{4}$$

$$\mu^N = x - \sigma^N \times \Phi^{-1}[F(x)] \tag{5}$$

In these expressions, x is the original non-normal variant, $\Phi^{-1}[.]$ is the inverse of the standard normal cumulative distribution (CDF), $F(x)$ is the original non-normal CDF evaluated at x, $\varphi^{-1}[.]$ is the probabilistic density function of the standard normal distribution, and $f(x)$ is the original non-normal probability density ordinate at point x. In the case of correlated non-normals, the ellipsoidal perspective (Fig. 1) and the constrained optimization approach still apply in the original coordinate system, with the exception of the fact that non-normal distributions are replaced by an equivalent normal hyper-ellipsoid, centered not at the original mean of the non-normal distributions, but at the equivalent normal mean μ^N.

Based on the reliability index, the probability of failure can be evaluated as follows:

$$p_f \approx 1 - \varphi(\beta) \tag{6}$$

In this expression, $\phi()$ refers the cumulative distribution function of the standard normal variable.

The first and third terms under the square root sign in Eq. (3) are equivalent standard normal vectors. Building on this, Low and Tang (1997b, 2007) and Low (2004) developed a practical FORM procedure using constrained optimization and proposed an efficient algorithm for evaluation using the varying dimensionless number n_i, recasting Eq. (3) as follows: