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Using inconsistency measures for estimating reliability

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ABSTRACT

Any decision taken by an agent requires some knowledge of its environment. Communication with other agents is a key issue for assessing the overall quality of its own knowledge. This assessment is a challenge itself as the agent may receive information from unknown agents. The aim of this paper is to propose a framework for assessing the reliability of unknown agents based on communication. We assume that information is represented through logical statements and logical inconsistency is the underlying notion of reliability assessment. In our context, assessing consists of ranking the agents and representing reliability through a total preorder.

The overall communication set is first evaluated with the help of inconsistency measures. Next, the measures are used for assessing the contribution of each agent to the overall inconsistency of the communication set. After stating the postulates specifying the expected properties of the reliability preorder, we show through a representation theorem how these postulates and the contribution of the agent are interwoven. We also detail how the properties of the inconsistency measures influence the properties of the contribution assessment. Finally we describe how to aggregate different reliability preorders, each of them may be based on different inconsistency measures.

1. Introduction

To be able to act or deliberate, any rational agent must acquire knowledge of its environment. It gets it by merging information provided by its own sensors and/or by merging information communicated by other agents. Merging basic information is a key issue for any agent as it is the underlying rational for decision making and it contributes to justify the agent's epistemic state. Techniques for merging raw information have been studied in an extensive way. These techniques usually assume that all information provided by the sources (i.e. agents) should be considered as a whole. Two different approaches have been studied: the first one considers sources in an equal way and has led to merging techniques such as majority merging, negotiation, arbitration merging or distance-based merging for solving conflict between contradicting information [21,8,27,9]. The second one distinguishes sources through a reliability criterion. Taking sources reliability into account provides rationales for discounting or ignoring pieces of information whose source is not considered as sufficiently reliable. Some promote a quantitative model of reliability: information sources are associated with a reliability level represented by a number used by the merging operator. According to the belief function theory, the reliability level of a source is a number between 0 and 1. This number is then used by the discounting rule in order to weaken the importance of infor-

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mation provided by this source [31]. Some others promote a qualitative approach to reliability and consider that information sources are ranked according to their reliability. This order or pre-order is then used by the merging operator. In [5], the author defines a merging operator which assumes that the sources are totally ordered: if s is said to be more reliable than s' and together provide contradicting information, then information provided by s is privileged; while information provided by s' which does not contradict information of s is also considered as acceptable. The same idea is followed by [22] for reasoning about more complex beliefs and in [24] for revising a belief base. All these works assume that the reliability of the sources is given as a parameter (quantitative or qualitative), they do not address the question of how to build up this reliability.

In this present paper our aim is to address the key question of how to build a reliability preorder of information sources, in a context where sources are unknown: no extra information about sources is available and information provided by the sources is only qualitative (i.e., statements). We adopt a qualitative point of view to represent reliability: the relative reliability of information sources is represented by a total preorder. We propose to consider a phase, before the information merging phase, during which information sources are observed in order to obtain a reliability preorder. The purpose of this phase is to analyze the inconsistency of information reported by the different sources w.r.t. some trusted knowledge.

Our main goal is thus to show that the relative reliability of information sources can be estimated from the inconsistency of reported information. Two different approaches can be followed. The first approach consists in using an ad-hoc model for reported information and in developing new inconsistency measures. The second approach consists in modeling reported information in a conventional way in order to use well known inconsistency measures.

In a recent paper [6], we followed the first approach. Reported information was modeled by pairs: $< agent, formula >$, $formula$ representing a piece of information communicated by $agent$. For instance, the set $\{< a, p >, < b, \neg p >, < b, q >\}$ represented the fact that agent a had reported p, agent b had reported $\neg p$ and had also reported q. The main notions (inconsistency, minimal inconsistent subsets, inconsistency measures...) available in the literature have been adapted to this model.

In this paper, our very motivation is to show an original application of inconsistency measures, i.e. *reliability estimation*. Our starting point is the existing inconsistency measures. Hereafter, we simplify the representation of reported information so that we can re-use these existing inconsistency measures for elaborating agent's reliability.

Our original contributions consist in (i) characterizing the individual contribution of each agent to the overall inconsistency of a set of reported information and (ii) introducing postulates which characterize the expected properties of the reliability preorder; Based on these axiomatic perspective on reliability assessment, we show (i) how the properties of the inconsistency measure influence the properties of the contributions measures and (ii) how postulates about reliability and properties of agent contribution are related through a representation theorem. Finally, we show how to aggregate several preorders possibly obtained through different inconsistency measures; namely we show how the overall aggregated preorder may satisfy the reliability postulates if the initial preorders also satisfy these postulates.

This paper is organized as follows. Section 2 and Section 3 introduce the main notions needed to assess reliability of agents. They introduce inconsistent communication sets and focus on measuring the inconsistency in communication sets. Based on the inconsistency measures, Section 4 shows how to assess the individual contribution of an agent to the overall inconsistency of a communication set. Contribution is first characterized in an axiomatic way and next two possible contribution functions instantiating the expected properties are detailed. Some implementation and complexity considerations are also addressed. Section 5 proposes a set of postulates which axiomatically characterize reliability preorders and show through two representation theorems how these postulates and the agent contributions are related. Still in Section 5, we present two possible solutions for building a reliability preorder compliant with these postulates. Section 6 considers the aggregation of several reliability preorders and shows how Arrow's condition for aggregation and our postulates interplay. Finally, Section 7 concludes the paper and discusses future work.

2. Inconsistent communication sets

This section introduces communication sets and focuses on their inconsistency.

2.1. Preliminaries

Let $\mathcal{L}$ be a propositional language of formulas defined over a finite set of propositional symbols $\mathcal{P}$, propositional constants $\top$, $\bot$ and the logical connectives $\wedge$, $\vee$, $\neg$. We use $p, q, r, ...$ to denote the propositional symbols and Greek letters $\phi, \psi, ...$ to denote formulas of the classical propositional logic defined over $\mathcal{L}$. An interpretation i is a total function from $\mathcal{P}$ to $\{0, 1\}$ from which an assignment to $\{0, 1\}$ is generated for all the formulas of $\mathcal{L}$ defined in the usual way of classical logic. As usual, $i(\top) = 1$ and $i(\bot) = 0$. Interpretation i is *a model* of formula ϕ iff $i(\phi) = 1$. *Tautologies* are formulas which are interpreted by 1 in any interpretation. We write $\models \phi$ when ϕ is a tautology. A formula is *consistent* iff it has at least one model. Otherwise it is *inconsistent*.

A *communication base*[1] K is a finite (possibly empty) set of formulas of $\mathcal{L}$. $At(K)$ denotes the set of propositional symbols appearing in formulas which belong to K. A communication base is *consistent* iff the conjunction of its formulas is

[1] This term corresponds to the term "knowledge base", widely used in the community of belief merging.

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