



# Combinatorial analysis of the subsystem reliability of the split-star network



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## ABSTRACT

The reliability of a generic system is the probability that the system is fully functional under a given suite of operational and environmental conditions over a given time period. In this paper, the subsystem reliability of the split-star network is derived using a combinatorial approach. Some numerical results of various simulation models are shown to validate the established analytical formulation.

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## 1. Introduction

In many distributed-memory multiprocessor computers, processing and memory elements are structured using various types of interconnection networks, and the design of the interconnection mechanisms significantly affects the practical performance [10]. When designing a high-performance computer system, one of the most important driving factors is to design a system that performs the intended function correctly, efficiently, and economically [16]. Despite designers' best efforts, a system will probably fail to fulfill what the users expect. Reliability evaluation is well known tool which addresses these concerns [1,14,15,17,22,23,29,30].

The reliability of a generic system is the likelihood that it performs its expected functions consistently well under the given conditions within a specified time interval [19]. The larger a system becomes, the more the failures the system could cause. Hence, maintaining high reliability is even more difficult for a large-scale network. A typical approach to computing reliability is to decompose the system into smaller ones using a graph-theory model in which nodes and/or links are assumed to fail independently with known probabilities. One series of related research topics addressing the reliability evaluation of hypercube-based networks uses diverse reliability measures such as network reliability, terminal reliability, task-based reliability, subsystem reliability.[11,20,26,27]:

- Network reliability: A system works as long as all nodes in the system are fault-free and connected.
- Terminal reliability: A system works as long as two given nodes are fault-free and connected.

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- Task-based reliability: A system works as long as a minimum number of connected nodes are available for the execution of a task.
- Subsystem reliability: A system works as long as some functional subsystems are available.

These reliability measures are disparate. For example, Najjar and Gaudiot [26] reported that the hypercube's fault resilience increases from 25% to 33% as its dimensionality increases from 4 to 10, and a 10-dimensional hypercube remains connected with a probability of 0.99 when about 33% of its nodes are incapacitated; Soh et al. [27] developed polynomial-time algorithms to compute the lower bounds on both network and terminal reliability for the hypercube; Das and Kim [11] gave an exact expression for sub-cube reliability under the random fault model, which assumes that there are  $f$  node faults randomly distributed in the hypercube. Moreover, Chang and Bhuyan [3] proposed a new model, namely the probability fault model, for evaluating the sub-cube reliability of the hypercube. In particular, they also proved that the probability fault model for computing sub-cube reliability is as equally accurate as the random fault model, but much more efficient. Thus, this paper concentrates on subsystem reliability under the probability fault model.

Reliability evaluation is important for large-scale multiprocessor systems. A classical real-world multiprocessor system is the Intel iPSC/860, which uses a hypercube to interconnect up to 128 microprocessors. Pleiades is NASA's state-of-the-art peta-scale supercomputer, in which thousands of nodes are connected in a partial hypercube topology [31]. In addition, Kuhn et al. [21] presented a churn-resistant peer-to-peer system whose overlay topology is a hypercube. In the practical applications of such supercomputers, the presence of node failures is inevitable so that the complete network turns out to be unavailable. Reliability plays a useful role in predicting how the performance declines under failure conditions. In addition, the subsystem reliability can effectively evaluate how likely a smaller fault-free network is to remain available in a damaged system.

The star-graph network [2] was proposed as an alternative to the hypercube. Although the hypercube has long been a popular interconnect, a comparative study confirmed the topological superiority of the star-graph network [13]. In particular, Fitzgerald and Latifi [15] modeled the assessment of sub-star reliability for the star-graph network, and Wu and Latifi [28] reported that the sub-star reliability of the star-graph network drops much more slowly over time than the sub-cube reliability of the hypercube when the two networks contain a similar number of nodes. The challenge of their studies comes from the fact that sub-stars in a star-graph network intersect with one another in a complex way. Wu and Latifi [28] proposed two schemes to analyze the sub-star reliability in the star-graph network under the probability fault model: the first scheme derives an upper bound to the sub-star reliability, and the second one calculates an approximate sub-star reliability by ignoring any intersection among the sub-stars. Recently, Lin et al. [23] also applied these two schemes to establish an upper bound on the subgraph reliability of the arrangement graph under the probability fault model. In addition, Zarezadeh and Asadi [30] turned their attention to modeling network reliability under a stochastic process for component failures.

Cheng et al. [7] proposed the split-star network as a variant of the star-graph network. In particular, a split-star network can be decomposed into two disjoint alternating group networks. Both the star-graph and alternating group networks are special members of the family of generalized arrangement graphs [12,18]. Therefore, the split-star network inherits the topological properties of the alternating group networks and arrangement graphs, and can be a good candidate for a multiprocessor interconnection. These networks have been the focus of many researchers' attentions [4–6,8,9,24,25]. In this paper, the subsystem reliability of the split-star network is analyzed by using a combinatorial approach. Similar to the study by Wu and Latifi [28], it is difficult to determine the exact subsystem reliability because the total number of subsystems grows quadratically with respect to network dimensionality. In contrast to the upper-bound estimation proposed in [28], a lower-bound formulation is a better alternative because the network will be at least as reliable as the lower-bound value. Thus, our reliability analysis includes both upper- and lower-bound estimations. Some numerical results are presented for a number of probabilistic models to validate the established analytic formulation.

The rest of this paper is structured as follows: Section 2 introduces the fundamentals of the probability fault model and topological properties of split-star networks. Section 3 is devoted to the evaluation of subsystem reliability with respect to the given split-star network. Section 4 presents some numerical results based on simulated data. Finally, our concluding remarks are drawn in Section 5.

## 2. Preliminaries

Throughout this paper, graphs (interchangeably, networks) are finite, simple, and undirected. An *undirected graph*  $G$  consists of a vertex set  $V(G)$  and an edge set  $E(G)$ , where  $V(G)$  is a finite set, and  $E(G)$  is a subset of the set of all unordered pairs of distinct elements in  $V(G)$ . In this paper, we use  $\{u, v\}$  to denote an unordered pair of two elements  $u, v$ . Two vertices  $u$  and  $v$  of  $G$  are *adjacent* if  $\{u, v\} \in E(G)$ . A graph  $H$  is a *subgraph* of  $G$  if  $V(H) \subseteq V(G)$  and  $E(H) \subseteq E(G)$ . For any nonempty subset  $S$  of  $V(G)$ , the subgraph of  $G$  *induced* by  $S$  is a graph whose vertex set is  $S$  and whose edge set consists of all the edges of  $G$  joining any two vertices in  $S$ .

### 2.1. Split-star network

Let  $n$  be any positive integer. A permutation of  $n$  distinct identifying codes  $c_1, c_2, \dots, c_n$  is an ordered sequence containing each code exactly once. Throughout this paper, we always use positive numbers from 1 to  $n$  to form the collection of  $n$



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