Commun Nonlinear Sci Numer Simulat 60 (2018) 184–202

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

A new effective approach for computation of reliability index in nonlinear problems of reliability analysis

Mohsen Ali Shayanfar[a,*], Mohammad Ali Barkhordari[b], Mohammad Amin Roudak[b]

[a] The Centre of Excellence for Fundamental Studies in Structural Engineering, Iran University of Science and Technology, Narmak, Tehran 16846, Iran

[b] School of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran 16846, Iran

ARTICLE INFO

Article history:
Received 15 December 2016
Revised 29 December 2017
Accepted 24 January 2018

Keywords:
Design point
Reliability index
Search direction vector
Step size

ABSTRACT

In reliability analysis, locating design point and computing reliability index is still a challenge. In this paper a simple and efficient reliability algorithm is proposed. The proposed algorithm tries to approach design point purposefully during the iterations. This iterative algorithm works based on the effective use of the represented search direction vectors with appropriate step sizes. In the first part of the proposed algorithm, a search direction vector is represented to locate the general vicinity of design point. In the second part, two search direction vectors are represented such that an effective search around design point is carried out. This search is actually like a damped oscillating movement around design point, from safe to failure domain and vice versa. By such a movement, the distance with design point is reduced until it is found. Generally, in representing the search direction vectors and step sizes of two parts of the proposed algorithm, the main focus is on creating a simple and fast movement towards design point. The proposed algorithm is implemented in several numerical nonlinear examples, among the commonly used examples of the literature, to show its strong performance.

1. Introduction

In the field of structural analysis, it is observed that material, load, and geometric details of structures are extremely tied to uncertainty. Therefore, this uncertainty should be involved in the structural analysis to reach more realistic results [1,2]. Structural reliability theory is the tool to bring the effect of uncertainty into the analysis procedure [3-5]. In this theory, the main goal is to find the probability of failure P_f. This probability can be derived as

$$P_f = \int_{g(X)<0} f_X(X)\mathrm{d}X \tag{1}$$

where $X=[X_1, X_2,\ldots,X_n]^T$ is the vector of random variables and $f_X(X)$ denotes joint probability density function (JPDF) of this vector. $g(X)$ is limit state function (LSF) and $g(X)<0$ defines failure domain. As it can be seen, dealing with the above multi-dimensional integral is a big computational challenging. This challenge becomes more severe in large and complex

* Corresponding author.
E-mail addresses: shayanfar@iust.ac.ir (M.A. Shayanfar), barkhordar@iust.ac.ir (M.A. Barkhordari), amin_roudak@civileng.iust.ac.ir (M.A. Roudak).

https://doi.org/10.1016/j.cnsns.2018.01.016
1007-5704/

structures, or in structures with low probability of failure and highly nonlinear LSFs. This difficulty is due to the existence of multiple integral and JPDF of random variables. The aforementioned computational challenge caused the development of other alternatives such as simulation methods [6-8] and approximation methods [9-12].

In simulation methods, Monte Carlo simulation (MCS) is a basic method. In this method, a sampling density function (SDF) is employed to generate random samples. Then LSF is evaluated for each sample. P_f in MCS is computed as the proportion of the number of samples corresponding to failure (negative value of LSF) to total sampling number. Importance sampling is a technique which focuses on the way samples are generated and actually tries to select a more efficient SDF [13-16]. Generally, simulation methods result in relatively accurate solutions, but they are time consuming and impractical in many cases [17-20].

Among approximate methods, the so-called first-order reliability method (FORM) is the most widely employed method. One of the most important basic method in this class was proposed by Hasofer and Lind [21]. They introduced the following transformation from X-space to U-space (in which mean and standard deviation of all variables are 0 and 1, respectively)

$$u_i = \frac{x_i - \mu_{X_i}}{\sigma_{X_i}} \tag{2}$$

where μ_{X_i} and σ_{X_i} are mean and standard deviation of ith random variable, respectively. According to their definition, the least distance of limit state surface from the origin of U-space is called reliability index denoted by β. Moreover, the point with this minimum distance is called design point or most probable point (MPP). Thus, in general, the calculation of reliability index can be formulated as the following constrained optimization problem

$$\begin{array}{ll} \text{minimize} & (U^T U)^{1/2} \\ \text{subject to} & G(U) = 0 \end{array} \tag{3}$$

where G denotes limit state function in U-space. Rackwitz and Fiessler [22] extended the Hasofer–Lind method to take the distribution of random variables into account. They suggested the following transformation, instead of the transformation of Eq. (2), for non-normal variables

$$u_i = \frac{x_i - \mu'_{X_i}}{\sigma'_{X_i}} \tag{4}$$

in which μ'_{X_i} and σ'_{X_i} are called mean and standard deviation of ith equivalent normal variable, respectively. These parameters are computed by

$$\sigma'_{X_i} = \frac{\varphi\left\{\Phi^{-1}\left[F_{X_i}(x_i^*)\right]\right\}}{f_{X_i}(x_i^*)} \tag{5}$$

$$\mu'_{X_i} = -\sigma'_{X_i}\Phi^{-1}\left[F_{X_i}(x_i^*)\right] + x_i^* \tag{6}$$

where φ and Φ are the probability density function (PDF) and cumulative distribution function (CDF) of standard normal distribution, respectively. $f_{X_i}(x_i^*)$ and $F_{X_i}(x_i^*)$ are respectively the PDF and CDF of the original variable X_i at x_i^* (i.e. at the ith component of design point in X-space). In many situations this method denoted by HL–RF (Hasofer, Lind–Rackwitz, Fiessler) converges faster than other methods. However, in the case of highly nonlinear LSFs, this method may converge slowly or even result in divergence.

In recent decades, many other researches have been conducted. Liu and Kiureghian [23] used the concept of merit function to monitor the convergence of the problem. Their method improved the stability of the original HL–RF method. Wang and Grandhi [24] introduced the concept of intervening variables and nonlinearity index to improve the performance of HL–RF. This index is itself computed through a sub-iterative process at each iteration. Elegbede [25] established the particle swarm optimization algorithm for computation of reliability index in the case of normal random variables. Santosh et al. [26] improved the performance of the modified HL–RF by selecting a suitable step size using Armijo rule. This imposes additional sub-iterations in each iteration. Yang [27] has analyzed several examples using a stability transformation approach. His algorithm converges very slowly in highly nonlinear problems. Gong and Yi [28] introduced a new step length parameter in the direction of gradient vector. When this parameter tends to infinity the method reduces to HL–RF. Gong et al. [29] proposed a non-gradient-based algorithm for computation of reliability index. Roudak et al. [30] proposed a robust two-parameter method to remove numerical instability of HL–RF. When these two parameters take two specific values, the method reduces to HL–RF. Besides, when only the first parameter takes a specific value, the method reduces to the method proposed by Gong and Yi (in [28]). Thus, this method is a generalization for HL–RF and the method proposed by Gong and Yi. In another study, Roudak et al. [31] introduced a moving fitness function and proposed a three-phase reliability algorithm. Shayanfar et al. [32,33] proposed two methods, for finding reliability index, by combining the concepts of approximate methods and sampling of simulation methods.

In this paper an efficient iterative algorithm for computation of reliability index is proposed. This algorithm moves towards design point in a purposeful way and in two parts. This purposeful movement makes the proposed algorithm reliable such that in extremely nonlinear cases it can result in convergence. The other important feature of the proposed algorithm is its simple implementation, without need to merit functions or a line search process at each iteration. Through various types of numerical examples, it will be shown that the simple and reliable movement of the proposed algorithm towards design point is provided using a small number of iterations.