A reliability estimation approach via Wiener degradation model with measurement errors

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**ABSTRACT**

This paper proposes a reliability estimation approach based on EM algorithm and Wiener processes by considering measurement errors. Firstly, the time-transformed Wiener processes are used to model the degradation process of the product, which simultaneously consider the temporal variability, unit-to-unit heterogeneity and measurement errors. In addition, we obtain the closed-form expressions of some reliability quantities such as reliability function and probability density function of the life. Moreover, the expectation maximization algorithm is adopted to estimate the model parameters effectively. Finally, a numerical example and a practical case study for LED lamps are provided to illustrate the effectiveness and superiority of the presented approach.

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1. Introduction

Reliability estimation based on degradation data has attracted a great deal of interest in reliability analysis and maintenance scheduling for highly reliable products [1–4]. Degradation data can provide much more reliability information for deteriorating products than the failure data because they can characterize the underlying failure processes [5–7]. Compared with the traditional failure data analysis, degradation analysis are often able to obtain more accurate reliability estimation under the circumstance of less testing units. The degradation evolution of many products arises in a stochastic way such as LED lamps, rotating machines, and bridge beams. Therefore, the use of stochastic processes can be helpful to effectively model the degradation process of the product and estimate the reliability quantities. Based on the reliability estimation results, preventive maintenance scheduling can be timely triggered to minimize the unscheduled downtime and reduce the cost of the whole life cycle [8,9].

The stochastic processes with independent increments, such as Wiener processes and inverse Gaussian processes, have been widely used in the degradation modeling of products. For the non-monotonic degradation process frequently encountered in practice, Wiener processes are a more appropriate choice since they not only have some favorable mathematical properties but also can model the non-monotonic degradation evolution. Whitmore and Schenkelberg in [10] applied a regular Wiener process to model the degradation process of a self-regulating heating cable. Tseng et al. in [11] further developed an integrated Wiener process to model the degradation process of LED lamps. These works only considered the effect

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https://doi.org/10.1016/j.amc.2017.09.020
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of the temporal variability associated with the degradation evolution of a product. However, the degradation processes of many products are often affected by the unit-to-unit heterogeneity and measurement errors. The unit-to-unit heterogeneity means that the degradation paths among the same batch of products can be quite different owing to certain discrepancy in raw materials or manufacturing processes, and the observed degradation data are generally contaminated by measurement errors because of imperfect instruments and random environments.

In the existing degradation modeling works based on Wiener processes, the unit-to-unit heterogeneity has been well characterized by assuming certain parameters in the degradation model follow certain distributions. Peng and Tseng in [12] presented a degradation modeling approach based on the Wiener process with linear drift by considering measurement errors. To capture the unit-to-unit heterogeneity among a batch of products, they assumed the drift parameter follows a normal distribution, and obtained an analytically tractable degradation model. Nevertheless, it is noteworthy that the effect of measurement errors on the life distribution was not considered in their approach, therefore the obtained life estimation results are approximation. Si et al. in [13] applied the same degradation model to characterize the degradation process of the gyro in an inertial navigation platform. This work took the effect of measurement errors on the life distribution into account, but ignored the effect of the unit-to-unit heterogeneity on the life estimation. Ye et al. in [14] used a time-transformed Wiener process model with measurement errors to analysis the wear problem of magnetic heads used in hard disk drives and LED degradation problem, in which the normal distribution is also used to describe the unit-to-unit heterogeneity. However, it is worth noting that the expresses of the life distribution are not given in their work. The closed-form expressions of the life distribution are quite valuable for support decision-related applications such as maintenance scheduling, logistic planning, etc. For these reasons, it is necessary to simultaneously consider the temporal variability, unit-to-unit heterogeneity and measurement errors to provide more accurate reliability estimation results for subsequent maintenance scheduling. In addition, the above works obtain the maximum likelihood estimation (MLE) of the model parameters only by the direct maximization of the likelihood function. However, the estimated results are unsatisfactory when considering unit-to-unit heterogeneity in the degradation modeling. The main reason lies in the fact that the likelihood function includes unobserved hidden variables. As such, we need to investigate more effective parameter estimation method to obtain more accurate reliability estimation.

Motivated by the aforementioned works, the objective of this paper is to develop a reliability estimation approach by using a time-transformed Wiener process with measurement errors. The main contributions of this paper are summarized as follows. Firstly, the temporal variability, unit-to-unit heterogeneity and measurement errors are simultaneously taken into account in the developed degradation modeling method. Secondly, we obtain an exact yet closed-form expressions of the reliability quantities, such as reliability function and probability density function of the life, which can provide valuable information for subsequent maintenance scheduling. Thirdly, the expectation maximization (EM) algorithm is effectively used to obtain the MLE of the unknown parameters since the likelihood function contains unobserved hidden variables.

The rest of this paper is organized as follows. Section 2 presents the degradation modeling principle based on a time-transformed Wiener process with measurement errors and the closed-form expression of the reliability quantities in terms of the presented degradation model, while the statistical inference based on the EM algorithm is developed in Section 3. Section 4 provides a practical case study to illustrate the effectiveness of the proposed approach. Section 5 concludes this paper.

2. Degradation modeling and reliability estimation

Let \( X(t) \) represent the underlying degradation evolution of a deteriorating system over time \( t \), which is widely modeled by Wiener processes [15–17]. As a general rule, the underlying degradation state cannot be observed directly due to the inaccuracy of the measurement. Therefore, the observed degradation data are often influenced by imperfect measurement, i.e., the measurement process of \( X(t) \) includes the measurement errors. To characterize the influence of the measurement errors, the observed degradation process can be expressed as

\[
Y(t) = X(t) + \varepsilon = X(0) + \beta \Lambda(t) + \sigma W(\Lambda(t)) + \varepsilon.
\]

(1)

where \( \beta \) and \( \sigma \) represent the drift parameter and diffusion parameter respectively. \( \Lambda(t) \) denotes the transformed time scale, which is generally a monotone function with \( \Lambda(0) = 0 \). \( W(\Lambda(t)) \) represents a nonstandard Brown motion process in the case of \( \Lambda(t) \neq t \), which describes the temporal uncertainty of the degradation process. \( \varepsilon \) denotes the measurement error, which is generally assumed to be normally distributed with zero mean and standard deviation \( \sigma \). In addition, \( \varepsilon \) is assumed to be \( t \)-independent with \( \beta \). Without loss of generality, the initial degradation state \( X(0) \) is assumed to be zero in the following. In the degradation model above, the drift parameter \( \beta \) generally represents the degradation rate of a system, and the diffusion parameter \( \eta \) has no definite physical meaning. Considering the randomness of the materials and operating conditions, there always exist the different degradation rates for diverse systems in engineering practice. As such, it is more suitable to incorporate unit-to-unit heterogeneity into the degradation modeling. A typical assumption is that the drift parameter \( \beta \) is regarded as a random parameter for describing unit-to-unit heterogeneity, and \( \sigma \) is considered to be a fixed parameter for depicting the degradation characteristic common to all systems within the population. In general, the drift parameter \( \beta \) is supposed to follow a normal distribution, which has been widely adopted by many studies [14,18,19].

In the following, we will illustrate how to obtain the reliability quantities in terms of the degradation model (1). A system is often regarded as a failure when the degradation path first reaches a predefined failure threshold \( \omega \). Therefore, the concept
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