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Influence of rock property correlation on reliability analysis of rock slope stability: From property characterization to reliability analysis

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Cohesion (c) and friction angle (ϕ) of rock are important parameters required for reliability analysis of rock slope stability. There is correlation between c and ϕ which affects results of reliability analysis of rock slope stability. However, the characterization of joint probability distribution of c and ϕ through which their correlation can be estimated requires a large amount of rock property data, which are often not available for most rock engineering projects. As a result, the correlation between c and ϕ is often ignored or simply assumed during reliability studies, which may lead to bias estimation of failure probability. In probabilistic rock slope stability analysis, the influence of ignoring or simply assuming the correlation of the rock strength parameters (i.e., c and ϕ) on the reliability of rock slopes has not been fully investigated. In this study, a Bayesian approach is developed to characterize the correlation between c and ϕ, and an expanded reliability-based design (RBD) approach is developed to assess the influence of correlation between c and ϕ on reliability of a rock slope. The Bayesian approach characterizes the site-specific joint probability distribution of c and ϕ, and quantifies the correlation between c and ϕ using available limited data pairs of c and ϕ from a rock project. The expanded RBD approach uses the joint probability distribution of c and ϕ obtained through the Bayesian approach as inputs, to determine the reliability of a rock slope. The approach gives insight into the propagation of the correlation between c and ϕ through their joint probability into the reliability analysis, and their influence on the calculated reliability of the rock slope. The approaches may be applied in practice with little additional effort from a conventional analysis. The proposed approaches are illustrated using real c and ϕ data pairs obtained from laboratory tests of fractured rock at Forsmark, Sweden.

1. Introduction

In rock engineering, the emergence of simplified reliability methods and Monte Carlo simulation has allowed rock engineers and practitioners to improve geotechnical analyses by the calculation of the probability of failure (P_f) in addition to the factor of safety requirements (e.g., Tamimi et al., 1989; Jimenez-Rodriguez et al., 2006). The calculations of P_f incorporate the inherent variability in geotechnical parameters into evaluation of stability of geotechnical structures (e.g., Phoon, 2017). In reliability analysis of rock slopes, parameters of shear strength of discontinuities (i.e., cohesion (c) and friction angle (ϕ)) are required (e.g., Jimenez-Rodriguez et al., 2006; Jimenez-Rodriguez and Sitar, 2007; Low, 2007; Li et al., 2011). During reliability analysis of rock slopes, c and ϕ are often treated as random variables, and it has been widely reported that there is site-specific correlation between c and ϕ (e.g., Fenton and Griffiths, 2003; Wu, 2013; Wang and Akeju, 2016). Site-specific correlation between two rock parameters (e.g., c and ϕ) in this study refers to a relationship between the rock parameters at a site, such that a variation in the values of one parameter (e.g., c) leads to variation in the values of the other parameter (e.g., ϕ). Evaluation of the exact reliability of rock slopes, therefore requires that the joint probability density function (PDF) of correlated rock parameters (i.e., c and ϕ) is known, which is needed to quantify the correlation between them (i.e., c and ϕ).

However, in most geotechnical engineering projects, the joint PDF is often unknown because of limited data from field and laboratory tests (e.g., Li et al., 2012; Wang and Akeju, 2016). As a result of limited data, c and ϕ are often modelled in reliability analysis

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using only their means and standard deviations without considering the correlation between them through their joint PDF, or at best, the correlation between c and ϕ is simply assumed (e.g., Jimenez-Rodriguez et al., 2006; Wang et al., 2013). Ignoring or simply assuming the correlation between c and ϕ in a reliability analysis may lead to underestimation or overestimation of P_f (e.g., Baecher and Christian, 2003; Low, 2007; Tang et al., 2013; Wu, 2013). For example, Wang and Aladejare (2016a) noted that if the correlation between random variables is not properly modelled in reliability analysis, the failure probability obtained from there reliability analysis might differ by orders of magnitude. While many investigators have contributed to the subject of reliability or failure probability of rock slopes (e.g., Low, 2007; Park and West, 2001; Duzgun et al., 2003; Hoek, 2007; Duzgun and Bhasin, 2009; Li et al., 2011; Lee et al., 2012), effort has not been directed towards quantification of the site-specific correlation between c and ϕ, or their influence on reliability analysis process. Hence, it is of practical interest to develop approaches to quantify site-specific correlation between c and ϕ, and to assess their influence on reliability analysis of rock slopes.

This study addresses these challenges through development of a Bayesian approach for rock property characterization and an expanded reliability-based design (RBD) approach for probabilistic analysis of rock slope stability, with a view to assessing the influence of correlation between c and ϕ on reliability of rock slopes. The Bayesian approach involves modelling of variability in c and ϕ, followed by the development of the joint PDF of c and ϕ using a Bayesian framework. The derived joint PDF of c and ϕ is then incorporated into Markov chain Monte Carlo (MCMC) simulation to generate a large number of sample pairs of c and ϕ. The large number of sample pairs generated using MCMC represents the site-specific joint distribution of c and ϕ from which joint and marginal probability distributions as well as the correlation between c and ϕ can be evaluated. The sample pairs of c and ϕ are used as input in expanded RBD of rock slopes, that formulates the design process as an expanded reliability problem in which Monte Carlo simulations (MCS) are used in the design. For illustration purpose, the proposed approaches are applied to quantify the joint distribution and correlation of c and ϕ, and to perform reliability analysis of rock slope to assess the influence of correlation during the determination of the maximum rock slope height for safe excavation at Forsmark, Sweden.

2. Bayesian approach for rock property characterization

Bayesian framework for rock property characterization can be used to obtain the joint PDF of c and ϕ for quantifying the correlation between them, even when only limited site observation data pairs of c and ϕ are available. Bayesian framework provides a logical vehicle to systematically integrate information from different sources, such as observation data and knowledge available prior to collection of site-specific observation data (i.e., prior knowledge) (e.g., Ang and Tang, 2007; Aladejare, 2016; Wang et al., 2016). The Bayesian framework formulates the characterization of joint PDF of c and ϕ as an inverse analysis problem, in which the observation data pairs of c and ϕ are used as inputs for an inverse analysis model to infer the joint distribution of c and ϕ of rock as the model output. In the next subsections, the formulation of joint PDF of c and ϕ and probabilistic characterization of the joint PDF formulated are presented.

2.1. Joint probability distribution of c and ϕ

Geotechnical materials like rocks are natural materials whose properties are affected by various factors during their formation process (e.g., Baecher and Christian, 2003; Sari, 2009; Wang and Aladejare, 2016b). Rock properties therefore vary spatially, and there may be correlation between a pair of properties at the same location, especially when they are obtained together through the same process as in the case of c and ϕ. The concept of modelling the uncertainty in a rock property as a random variable with a probability distribution (e.g., normal or lognormal) has been previously extended to two correlated rock properties (e.g., Wang and Aladejare, 2016a). Previous studies have modelled c and ϕ as normal random variables (e.g., Low, 2007; Wang and Akeju, 2016), therefore, in this study, c and ϕ are modelled as normal random variables with means μ_c and μ_ϕ, respectively; and standard deviations σ_c and σ_ϕ, respectively. The correlation between c and ϕ is quantified by coefficient of correlation, ρ, which is also treated as a random variable in the Bayesian approach.

The site-specific joint probability distribution of c and ϕ can be obtained using the concept of mixture model, which considers the site-specific joint probability distribution as a weighted summation of various component density functions with different distribution parameters (e.g., Wang et al., 2015). A bivariate normal distribution is used to model the site-specific joint probability distribution of c and ϕ from available site observation data. In the concept of the mixture model, a bivariate normal distribution function is the component density function used in this study for constructing the site-specific joint probability distribution of c and ϕ from limited site-specific data. As the site-specific joint probability distribution is a weighted summation of various bivariate normal distributions (i.e., the component density function) with different combinations of parameters (i.e., means and standard deviations of c and ϕ as well as correlation coefficient), it is not necessarily a bivariate normal distribution (e.g., Wang et al., 2015). Distribution parameters of a bivariate normal distribution include μ_c, μ_ϕ, σ_c, σ_ϕ and ρ, which are needed to completely depict the joint distribution of correlated c and ϕ at a site. Both site-specific observation data and knowledge or information available prior to collection of site-specific observation data (i.e., prior knowledge) are used to estimate the distribution parameters μ_c, μ_ϕ, σ_c, σ_ϕ and ρ. The prior knowledge used in this study are the typical ranges of μ_c, μ_ϕ, σ_c, σ_ϕ and ρ available in geotechnical literature. For a given set of prior knowledge and site observation data pair of c and ϕ, there are many possible combinations of μ_c, μ_ϕ, σ_c, σ_ϕ and ρ values. Each possible combination of μ_c, μ_ϕ, σ_c, σ_ϕ and ρ has its corresponding occurrence probability, which is defined by a joint conditional PDF $P(\mu_c, \mu_\phi, \sigma_c, \sigma_\phi, \rho \mid Data, Prior)$. The *Data* denotes the site-specific observation data pairs of c and ϕ obtained during field and/or laboratory tests, while *Prior* denotes prior knowledge. Using the theorem of total probability (e.g., Ang and Tang, 2007), the joint PDF of c and ϕ for a given set of prior knowledge and site observation data is expressed as:

$$P(c,\phi \mid Data, Prior) = \int_{\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho} P(c,\phi \mid \mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho) \times P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho \mid Data, Prior)\, d_{\mu_c} d_{\mu_\phi} d_{\sigma_c} d_{\sigma_\phi} d_\rho \quad (1)$$

where $P(c,\phi \mid \mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)$ is the joint conditional PDF of c and ϕ for a given set of μ_c, μ_ϕ, σ_c, σ_ϕ and ρ. Since the joint distribution of c and ϕ is modelled using a bivariate normal distribution, the joint conditional PDF $P(c,\phi \mid \mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)$ is expressed as: