A one-level limit order book model with memory and variable spread

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Abstract

Motivated by Cont and de Larrard (2013)’s seminal Limit Order Book (LOB) model, we propose a new model for the level I of a LOB in which the arrivals of orders and cancellations are still assumed to be mutually independent, memoryless, and stationary, but, unlike the aforementioned paper, the information about the standing orders at the opposite side of the book after each price change and the arrivals of new orders within the spread are incorporated. Our main result gives a diffusion approximation for the mid-price process, which sheds further light on the relation between the mid-price behavior at low frequencies and some LOB features not considered in earlier works. To illustrate the applicability of the proposed framework, we also develop a feasible method to compute several quantities of interest, such as the distribution of the time span between price changes and the probability of consecutive price increments conditioned on the current state of the book. These LOB model features are relevant in many applications such as high frequency trading and intraday risk management. The proposed method is also used to develop an efficient simulation scheme for the price dynamics, which is then applied to assess numerically the accuracy of the diffusion approximation.

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1. Introduction

Most modern financial exchanges make use of electronic communication networks that implement a continuous double auction trading mechanism. Two types of orders are available in these trading platforms: limit and market orders. Broadly, a bid (ask) limit order specifies a price at which a trader is willing to buy (sell) a determined number of shares of an asset, while a market order is a request to immediately buy or sell a specified number of shares at the best available prices out of all the active limit orders. Other than limit orders and market orders, cancellation of limit orders is another common operation. The Limit Order Book (LOB) aggregates all the outstanding limit orders at any given time and offers a unique glimpse into the forces and rules of price formation of an asset.

With the advent of real-time and historical LOB data, LOB modeling has received substantial attention in recent years. We refer to Gould et al. [9] for a nice survey of the extensive literature on the subject, which, among the most relevant to our work, includes Luckock [13], Kruk [12], Smith et al. [19], Rosu [17], Cont et al. [7], Abergel and Jedidi [1], Cont and Larrard [6], and Cont and Larrard [5]. The different models proposed so far obviously vary in complexity and detail depending on the applications being considered. In this work we present a new model for the dynamics of the best bid and ask levels of a LOB, which not only is able to incorporate several real features of LOB dynamics, but also is tractable enough for us to achieve two important objectives. Firstly, we are able to characterize the coarse-grain dynamics of the asset’s price process by establishing a diffusion approximation for it. More concretely, we prove that the mid-price process of the asset, properly scaled in time and space, converges to a Brownian motion with drift. This type of scaling limit enables us to connect the features of the process at lower frequencies (say, minutes, hours, or days) to the statistical properties of events taking place at the millisecond scale. A classical application of this is to analyze the behavior of the asset’s volatility as a function of both the intensities at which LOB events take place and a measure of the LOB’s depth. Secondly, we are capable of computing several quantities of interest such as the distribution of the time span between price changes and the probability of consecutive price increments conditioned on the current state of the book. These LOB model features are relevant in many applications, but in particular, in high frequency algorithmic trading and intraday risk management.

Our main inspiration for the present work is drawn from [6]’s seminal work, where a Markovian model is proposed for the dynamics of the LOB’s level I (i.e., the limit orders with the best prices to sell or buy the asset). There are two motivating factors for only considering the level I and not the entire book. Firstly, the asset’s price is only determined by the level I and, secondly, the information contained in the level I is key for many high-frequency trading strategies and problems. By imposing a Poissonian order flow and some symmetry conditions on the shape of the order book, Cont and Larrard [6] established the diffusion approximation for the mid price process mentioned in the previous paragraph. Unfortunately, the results in [6] required several strong assumptions, the most important of which are:

(i) a constant volume for all type of operations: market, limit, and cancellations;
(ii) a constant spread of one tick between the best ask and bid prices at all times;
(iii) constant parallel price shifts of one tick after each depletion of a level I queue;
(iv) loss of “memory”, in the sense that, after each level I queue depletion, the information on the remaining limit orders at the side which was not depleted is reset.

The previous assumptions are, of course, idealizations. For instance, the assumption of constant spread is generally reasonable for “large” tick assets, in which the tick size is comparable
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