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Optimal Current Trajectories for Power Converters with Minimal Common Mode Voltage

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Abstract: This article addresses the topic of computing optimized pulse patterns with common mode voltage constraints. The main thrust is to obtain tractable reformulations of the CMV constraints in the frequency domain in order to avoid complex mixed time-frequency formulations. The resulting optimization problem is a nonlinear one for which efficient numerical solvers are readily available. Moreover, we provide an algorithmic way of reducing the conservatism in the reformulated problem and validate our method with numerical illustrations that highlight the benefit of the proposed approach.

Keywords: inverter drives, optimal trajectory, nonlinear optimization, constraints

1. INTRODUCTION

Optimized pulse patterns (OPPs) or optimized Pulse Width Modulation (PWM) based schemes have a long history in the field of control of power converters, see, for example Buja (1980); Holtz and Qi (2013) and references therein. OPPs are extensively used since they tend to provide optimal current Total Demand Distortion (TDD) at the output of multilevel inverters for a fixed switching frequency. This latter property, i.e., fixed switching frequency, makes the usage of OPPs in the control of power converters even more attractive, as it limits the switching losses and hence improves the overall efficiency.

Pulse width modulation techniques for controlling power converters have been extensively studied, see Holmes and Lipo (2003) for an authoritative reference on the analysis of PWM methods. Optimal PWM methods or equivalently OPP methods are a subclass of PWM methods in which the switch signals are computed offline, based on a certain performance criterion - typically minimal current TDD - and a set of constraints on the fundamental component as well as the individual harmonics. In particular, Selective Harmonic Elimination (SHE) is one technique in which the switching angles are designed so that the maximal possible number of harmonics is eliminated, and hence the current TDD may be improved in this way Fei et al. (2009); Agelidis et al. (2008); Chiasson et al. (2008). There have been also many proposals on computing OPPs without SHE, for example in Meili et al. (2006). Control methods of power converters using OPPs as the main ingredient have appeared in Geyer et al. (2010, 2012); Holtz and Oikonomou (2007); Holtz and Beyer (1995, 1991); Rathore et al. (2013).

Computing OPPs *with* limited Common Mode Voltage (CMV) is a topic that has been rarely addressed, although it is of high importance. OPPs with reduced CMV lead to a better lifetime of the electric machine as they reduce the stress on the machine insulation. This extra benefit comes at the expense of an extra constraint to be incorporated into the optimization problem that generates the OPPs; this constraint is of a nonlinear nature and is a time-domain expression. In this article, we focus on generating OPPs for L-level converters with the CMV constraint and obtain tractable reformulations of the CMV constraint. These reformulations of the CMV constraint allow us to use either the Fourier coefficients that represent the OPPs for any L-level converter or the switching angles in the special case of 3-level converters.

This paper is organized as follows. Section 2 presents the problem of designing OPPs with minimal current TDD and a constraint on the CMV. We present various convex reformulations of the CMV constraint in the frequency domain in Section 3. We then present numerical results in Section 4, illustrating the conservatism of the various constraint reformulations. Finally, we conclude in Section 5 and provide directions for future work.

2. PROBLEM FORMULATION

Consider an L-level converter (Figure 1) operated using OPPs. Referring to Figure 2, denote by $v(t) = v(\omega_1 t)$ an OPP waveform, where $\omega_1 = 2\pi f_1 = \frac{2\pi}{T_1}$ and f_1 is the fundamental frequency. Without any loss of generality, we shall take $\omega_1 = 1$; however, the results in the paper remain valid for any choice of ω_1. The signal $v(t)$ is described by a sequence of switching angles $\bar{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_N]$ and the corresponding jumps in the voltage levels $\bar{f} = [f_1, f_2, \cdots, f_N]$, where

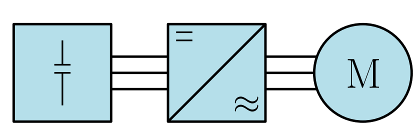

Fig. 1. A setup in which an inverter unit converts DC to AC signals to drive an electric machine.

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$$f_i = \begin{cases} +1, & \text{for a rising edge,} \\ -1, & \text{for a falling edge,} \end{cases} \tag{1}$$

and N is the so-called pulse number. We shall restrict our attention to OPPs with the following three properties.

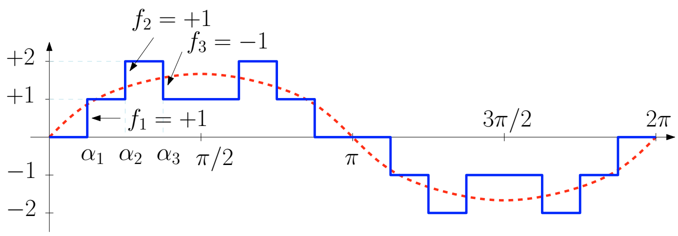

Fig. 2. An example of a pulse pattern $v(t)$ with $N = 3$ for a 5-level converter.

P1. *Quarter wave symmetry*: the following two conditions hold $v(t) = v(\pi - t)$, $\forall t \in \left[0, \frac{\pi}{2}\right]$ and $v(t) = -v(2\pi - t)$, $\forall t \in [0, \pi]$.

P2. *Three phase symmetry*:

$$\begin{aligned} v_a(t) &= v(t), \\ v_b(t) &= v\left(t - \frac{2\pi}{3}\right), \\ v_c(t) &= v\left(t + \frac{2\pi}{3}\right). \end{aligned} \tag{2}$$

P3. $v_a(t) \geq 0, \forall t \in [0, \pi/2]$.

Given the quarter wave symmetry, property (P1) and the fact that only single level jumps are allowed in the OPP, i.e., (1) holds, the Fourier series expansion of a single phase OPP is given by

$$v(t) = \sum_{k=1,3,5,\cdots} \hat{v}_k \sin(kt), \tag{3}$$

where $\hat{v}_k$ is the amplitude (Fourier coefficient) of k-th voltage harmonic given by

$$\hat{v}_k = \frac{4}{\pi} \frac{u_d}{(L-1)} \frac{1}{k} \sum_{i=1}^{N} f_i \cos(k\alpha_i), \tag{4}$$

where u_d is the full DC link voltage in an L-level converter. The derivation of (4) may be found in the Appendix of Hokayem et al. (2017).

2.1 Performance Index

Assuming that the OPPs are used to drive a machine with a Y-connection, then the 3 phase currents are driven by the voltages $v_{a0} = v_a - v_0$, $v_{b0} = v_b - v_0$ and $v_{c0} = v_c - v_0$, where

$$v_0(t) = \frac{1}{3}\left(v_a(t) + v_b(t) + v_c(t)\right) \tag{5}$$

is the common mode voltage (CMV) at the star point connection of the machine. Since v_0 contains all triple harmonics, these harmonics are absent from the current signals. Accordingly, we can define the current TDD as

$$TDD_i = \frac{1}{I_{rat}} \sqrt{\sum_{k \in \mathbb{H}_{TDD}} \left(\hat{i}_k\right)^2}, \tag{6}$$

where I_{rat} is the rated peak value of current, and $\mathbb{H}_{TDD} := \{5, 7, 11, 13, \cdots\}$. Assuming that the response of the induction machine at harmonics $\hat{v}_k$ beyond the fundamental can be modeled as a pure leakage inductance l_σ, i.e., the resistance is negligible, the current harmonics can be approximated by $\hat{i}_k \approx \frac{\hat{v}_k}{k l_\sigma}$. Upon substituting this expression into (6), we obtain the following expression $TDD_i \approx \frac{1}{l_\sigma I_{rat}} \sqrt{\sum_{k \in \mathbb{H}_{tdd}} \left(\frac{\hat{v}_k}{k}\right)^2}$ $= \frac{1}{l_\sigma I_{rat}} \frac{4u_d}{\pi(L-1)} \sqrt{\sum_{k \in \mathbb{H}_{tdd}} \left[\frac{1}{k^4}\left(\sum_{i=1}^{N} f_i \cos(k\alpha_i)\right)^2\right]}$. The nonlinear performance index TDD_i is to be minimized to obtain the optimal sequence $\bar{f}^* := [f_1^*, f_2^*, \cdots, f_N^*]$ and the corresponding set of optimal switching angles $\bar{\alpha}^* := [\alpha_1^*, \alpha_2^*, \cdots, \alpha_N^*]$, under a specific set of constraints that are discussed next.

2.2 Constraints

Foremost, one is interested in keeping the fundamental component of the Fourier series at a specific level (the modulation index m), i.e.,

$$\hat{v}_1 = m. \tag{7}$$

Another constraint may be related to the amplitude of individual harmonics in relation to the fundamental. This can be posed as

$$|\hat{v}_k| \leq \rho_k |\hat{v}_1| \tag{8}$$

where ρ_k's are some nonnegative factors. The constraint (8) is very useful when dealing with rectifier units as it allows the satisfaction of grid codes in terms of harmonic emissions into the grid.

One may also be interested in enforcing a constraint pertaining to the minimal separation between any two consecutive switches over a single phase, i.e.,

$$\begin{cases} 0 \leq \alpha_1 \leq \cdots \leq \alpha_N \leq \frac{\pi}{2}, \\ \alpha_{i-1} + \delta \leq \alpha_i, \quad \forall i \in \{2, \ldots, N\}, \end{cases} \tag{9}$$

where δ is the minimal separation between any two consecutive switching angles.

Assuming that the three phase symmetry (2) holds, then the CMV (5) can be written as

$$v_0(t) = \sum_{k \in \mathbb{H}_0} \hat{v}_k \sin(kt), \tag{10}$$

where $\mathbb{H}_0 = \{3, 9, 15, 21, 27, \cdots\}$ is the index set of the tripplen harmonics (multiples of 3 that are not even) that are present in the CMV; this fact is shown in the Appendix of Hokayem et al. (2017). The constraint on the CMV can now be defined as

$$|v_0(t)| = \left|\sum_{k \in \mathbb{H}_0} \hat{v}_k \sin(kt)\right| \leq \gamma, \tag{11}$$

where γ represents the maximally allowed bound on the CMV. Interesting to note, that due to the switched nature of the OPPs, the resulting CMV can only be equal to 0 or an integer multiple of $\frac{1}{3}$.

It is important to note here that none of the constraints (9), (7), and (8) pose changes to the forthcoming discussion in the paper, as the discussion pertains mainly to the approximation of the CMV constraint (11).

2.3 Optimization Problem

The resulting optimization problem for generating OPPs with minimal current TDD and limited CMV is given by