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Modeling and Stability of Microgrids with Smart Loads

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Abstract: A demand-side technology called "electric springs" was recently proposed to stabilize the future smart grid subject to large penetration of renewable energy sources. Based on power flow and basic circuit theory, this paper establishes a comprehensive microgrid model that consists of energy sources interfaced via inverters and smart loads for which electric springs are installed. Modeling it as a controlled voltage source, we can design the high-level control for the electric spring such that the load bus dynamics can be shaped to have desired frequency and voltage dynamics. We conduct the small-disturbance stability analysis on the established microgrid model where both generation and load buses implement frequency and voltage droop control. Finally, a graph-theoretic like stability condition is obtained.

Keywords: Microgrid, Smart Load, Electric Spring, Network-preserving Model, Network Topology.

1. INTRODUCTION

Due to the stochastic and intermittent nature, the integration of renewable energy into power systems such as microgrids (Lasseter (2002)) causes fluctuations in power flows, thus deteriorating the quality of power delivery and potentially leading to system instability. A demand-side control scheme was proposed in Westermann and John (2007); Zhao et al. (2014) to mitigate the mismatch between generation and demand by making the load demand follow the power generation. This technology usually requires additional controllability in load buses. Recently, a new smart grid technology called "electric springs" (ES) was proposed in Hui et al. (2012) to enable the demand-side control scheme at low cost and in a way less intrusive to the consumer in contrast with other demand-side response technologies such as energy storage systems used for the compensation and on-off control of loads. ES is a class of power electronic devices and can be ideally regarded as a current-controlled AC voltage source. Installed in series with the non-critical load, ES and the noncritical load together act as a smart load whose power consumption can be boosted and shedded when needed.

A few applications of ES at the distribution level of traditional power systems have been reported in Lee and Hui (2013); Tan et al. (2013); Luo et al. (2015) for achieving voltage stabilization and active power factor compensation. ES has also proven its utility in microgrids for the purpose of mitigating voltage and frequency fluctuations caused by the penetration of renewable energy (see e.g., Chen et al. (2015); Yan et al. (2015). In most of the existing work, the working principle and effectiveness of the ES are mainly demonstrated and validated using experiments and numerical simulation, while the theoretical analysis still remains on steady-state analysis and relatively simple system setups such as a single bus fed with a constant voltage source. A comprehensive microgrid model that is composed of distributed energy sources and smart loads in a power network has still not been presented yet.

In microgrids, the conventional power generators based on synchronous machines become less encountered, while the distributed energy sources interfaced via power electronic devices are more prevalent. As a result, the dynamics of microgrids are determined by the control logic of the power electronics and dynamical behaviors of smart loads rather than electro-mechanical dynamics of machines in traditional power systems. Consequently, the stability of microgrids particularly in islanded mode (isolated from the main grids) needs to be examined differently. In principle, there are two types of stability analysis considered in the literature, namely small-disturbance and transient stability analysis. Small-disturbance stability analysis is concerned with power system stability when subjected to fluctuations of small magnitude, while transient stability analysis is related to stability properties under large and sudden disturbances. The existing work mainly focuses on the stability analysis of microgrids without smart loads. The small-disturbance frequency stability of microgrids was studied in Song et al. (2015); Simpson-Porco et al. (2013a), while the transient stability was studied in Zhu and Hill (2016). The feasibility of reactive power flow and voltage stability of microgrids in a particular structure were investigated in Simpson-Porco et al. (2013b) under angle and voltage decoupling approximation.

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In this paper, we will derive the model for smart loads and then obtain the comprehensive model for microgrids composed of smart loads and distributed energy source equipped with inverters. Incorporating the model for smart load and generation buses, we formulate the frequency and voltage dynamics for microgrids. The comprehensive model can provide a more clear understanding of how a system component reacts to the variation of other components somewhere else in the system through power flow. Moreover, we will show the control degrees of freedom of ES allow the load bus dynamics to be shaped as desired provided that some local measurements are available. With the newly obtained microgrid model, a small-disturbance stability analysis for the overall system is provided compared with the existing work that deals with the frequency stability in Simpson-Porco et al. (2013a); Song et al. (2015) or voltage stability in Simpson-Porco et al. (2013b) separately.

The remainder of this paper is organized as follows. Section 2 first introduces the basics of nonlinear power flow, reviews models for energy sources with inverters in microgrids and presents the problem to be studied. In Section 3, we will obtain the model of smart loads and show that the ES can be used as a smart device to shape the load bus dynamics. In Section 4, we introduce a class of the load-side droop controllers and implement them using ES. Then, we conduct the small-disturbance stability analysis on the derived system and provide a graph-theoretic stability condition. The paper is concluded in Section 5.

Notations. x^* is the complex conjugate of the vector x and $j = \sqrt{-1}$ is the imaginary unit. The vector e_n is a column vector of dimension n with all elements being 1 and 0_n is zero column vector of dimension n. I and O are the identity and zero matrices with proper dimensions that are inferred from the context. For a symmetric matrix A, $A > 0$ and $A < 0$ mean A is positive and negative definite, respectively. $A \geq 0$ and $A \leq 0$ mean A is semi-positive definite and semi-negative definite, respectively.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Preliminaries

In general, a microgrid consists of a group of distributed energy sourced interfaced by inverters and loads. To facilitate the representation, associate the microgrid with a connected and undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, \cdots, n\}$ is the node set representing buses and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set describing the electric lines connecting buses. $(i, k) \in \mathcal{E}$ if bus i and bus k are connected. $\mathcal{V} = \mathcal{V}_S \cup \mathcal{V}_L$ where $\mathcal{V}_S$ and $\mathcal{V}_L$ are node sets for energy sources and loads, respectively. $\mathcal{N}_i \subset \mathcal{V}$ represents the set of buses to which bus i is connected. $y_{ik} = g_{ik} + jb_{ik}$ is the admittance of the electric line connecting bus i and bus k where g_{ik} and b_{ik} are line conductance and susceptance, respectively and $y_{ik} = 0$ if bus i and bus k are not connected. $Y \in \mathbb{C}^{n \times n}$ is the admittance matrix whose elements are defined as

$$Y_{ik} = \begin{cases} y_{ii} + \sum\limits_{l \in \mathcal{N}_i} y_{ik}, & i = k \\ -y_{ik}, & i \neq k \end{cases} \quad (1)$$

where y_{ii} is the self-admittance of bus i. Let $Y = G + jB$ where G is the conductance matrix and B is the susceptance matrix. In this paper, we assume the electrical lines are purely inductive and self susceptance is zero, namely $G_{ik} = 0$ and $B_{ik} \leq 0$, $\forall i, k = \{1, \cdots, n\}$ and thus $G = 0$.

Associated with each bus is the voltage phasor $E_i = V_i e^{j\theta_i}$ where $V_i > 0$ is the voltage magnitude and θ_i is the phase shift of the voltage phasor. The linear current-voltage relation is described as $I = YE$ where $I \in \mathbb{C}^n$ and $E = \text{col}(E_1, \cdots, E_N)$ are vectors of nodal current injections and voltages. The apparent power delivered is $S_e(V, \theta) = P_e(V, \theta) + jQ_e(V, \theta) = \text{diag}\{E\}(YE)^*$ where P_e and Q_e are vectors of active and reactive power delivered by corresponding buses, respectively. The active and reactive power flow can be expanded elementwise with $P_{e,i}(V, \theta)$ and $Q_{e,i}(V, \theta)$ as follows,

$$\begin{aligned} P_{e,i}(V, \theta) &= -V_i V_k B_{ik} \sin(\theta_i - \theta_k), \\ Q_{e,i}(V, \theta) &= V_i^2 B_{ii} + V_i V_k B_{ik} \cos(\theta_i - \theta_k), \ i \in \mathcal{V} \end{aligned} \quad (2)$$

Note that the positive sign of $P_{e,i}$ and $Q_{e,i}$ means bus i deliveries the power, while the negative sign means absorbs power.

2.2 Models with Energy Sources and Passive Loads

The distributed energy sources in microgrids are usually fed via inverters which implement frequency and voltage regulation controllers. Droop control is a common decentralized control strategy to achieve active power sharing in Simpson-Porco et al. (2013a) and reactive power sharing in Simpson-Porco et al. (2013b) at generation side for the islanded microgrids. The frequency droop controller is implemented such that the frequency deviation is proportional to the power delivery offset from the setpoint, which is described by $\omega_i - \omega_i^* = -n_i(P_{e,i} - P_i^*)$ where ω_i^* and P_i^* are the nominal system frequency and power delivery setpoint, and n_i is the droop coefficient. It is rewritten as

$$D_i \dot{\theta}_i = P_i^* - P_{e,i}, \ i \in \mathcal{V}_S \quad (3)$$

where $\dot{\theta}_i = \omega_i - \omega_i^*$ is the deviation from the nominal frequency and the constant $D_i = 1/n_i$ is referred to as the (inverse) droop coefficient. Note that the energy sources interfaced via PV inverters that perform maximum power point tracking and in series with a local frequency-dependent load can also be described by (3), while the maximum power point P_i^* might vary with weather conditions.

When the time scale between internal dynamics of inverters and higher-level control loop are considered to be separated, the inverter is approximated as a controllable voltage source with voltage dynamics $\tau_i \dot{V}_i = u_i$ where u_i is the control input to be designed and τ_i is a time constant related to delays in sensing and actuation process (see Simpson-Porco et al. (2013b)). The conventional voltage droop control in Zhong and Hornik (2012) specifies $u_i = -(V_i - V_i^*) - m_i(Q_{e,i} - Q_i^*)$ where V_i^* and Q_i^* are the nominal bus voltage and reactive power setpoint, and $m_i > 0$ is the droop coefficient. The closed-loop voltage dynamics become

$$\tau_i \dot{V}_i = -(V_i - V_i^*) + m_i(Q_i^* - Q_{e,i}), \ i \in \mathcal{V}_S. \quad (4)$$