Matching auction with winner's curse and imperfect financial markets

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ABSTRACT

This paper explains how and why the Matching Auctions work better with Imperfect Financial Markets. We show that an efficient outsider can obtain a "good" project even if the insider has informational advantage.

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1. Introduction

It is common practice that local and state governments prefer to give local businesses advantage over outsiders with the intention to ensure the best service *and* support the local economy. Procedures like *Matching Auctions* or a *right of first refusal* are common.[1] However, local businesses are better informed about local projects which creates asymmetry between insiders and outsiders. The consequence is that the outsiders are tamer in bidding, if at all, knowing that they will get business from the locals if they bid too high and will suffer from the *Winner's Curse*.[2] If they bid less than the project can promise, then the locals will match such a bid.

Interestingly, the current difficult financial times can actually help efficient outsiders to obtain local businesses from inefficient local companies. In this paper, we present a simple model which shows that the effect of *Imperfect Financial Markets* – when local companies have problems obtaining enough funds to match an outsider's bid – can outweigh the *Winner's Curse* effect.

There are three main factors in the model. First, the probability that the project is "good" has to be high enough. Since both the

outsider and the insider have asymmetric information, in order to make a bid the outsider has to be confident enough that the project is "good." Then, the cost of making a bid has to be low enough. If the cost is very high, the project may not compensate for it. Finally, financial markets have to be *imperfect*. This means that for one reason or another the insider sometimes cannot afford matching the outsider's bid even if he wants to. If financial markets are perfect, the outsider gets only "bad" projects (adverse selection), and therefore does not make a bid because of the *Winner's Curse*.

One of the applications of the model is in takeovers, in which one of the bidders knows the value of the object and the other bidders do not.[3] The well-known result is that the outcomes of standard auctions are highly sensitive to small asymmetries between bidders in (almost) common value settings. Many examples show that the bidder who knows the value gets an object at a low price because of the Winner's Curse (see Glaxo's takeover bid for the Wellcome Drugs company in Klemperer (1998, p. 763) and Huizenga matched bid for the Miami Dolphins football team in Nalebuff and Brandenburger (1996, pp. 174–175), among others). The crucial assumption is that the financial markets are perfect. However, if a bidder knows the value but cannot borrow money for matching the current bid, the outcome of such an

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[1] For Matching Auctions, see Riley and Samuelson (1981); for the right of first refusal, see Nalebuff and Brandenburger (1996).

[2] More about Winner's Curse can be found in Thaler (1988, 1992).

[3] For the Takeover games, see Bazerman and Samuelson (1983), Holt and Sherman (1994), Charness and Levin (2009) among others. Note that the markets are perfect in the Takeover game.

auction may be quite different from the prediction of the theory based on the assumption that financial markets are perfect.

2. The model: Matching Auction

Suppose that there are two risk-neutral buyers: *Insider* and *Outsider* in the Matching Auction. The seller has an object for sale and uses the following procedure: first, the *Outsider* submits a bid. Then the *Insider* observes this bid and can match it. If the *Insider* matches the bid, he obtains the object for the matched price. If the *Insider* does not match the bid, the *Outsider* obtains the object for the submitted price.

Suppose that the value of the object can be either "bad" or "good" and the *Insider's* values in these cases are 0 or 1, and the efficient *Outsider* obtains values 0, or $(1 + \alpha)$ in these cases, where $\alpha \geq 0$. We assume that the *Insider knows* the value and the *Outsider* only knows that the value is "good" with probability $0 \leq q \leq 1$, and "bad" with probability $1 - q$.

We assume that financial markets are imperfect and the *Insider* may have a problem with matching an outside bid. The *Outsider* knows that if her bid, t, is in the interval $(0, 1]$, the *Insider* cannot match it (even if the *Insider* wants to) with probability $0 \leq F(t) \leq 1$, because of imperfection in financial markets.

We assume that the *Insider* will match the outside bid if the object is "good" and the *Insider* can afford it (this happens with probability $1 - F(t)$). If the *Insider* does not match the outside bid, the *Outsider* gets the object.

The *Outsider* faces the following problem: she loses bid t if the object is "bad," which happens with probability $1 - q$. However, the *Outsider* can get the "good" object and obtain $((1 + \alpha) - t)$ if the *Insider* cannot afford to match bid t. It happens with probability $qF(t)$. Finally, the *Outsider's* bid will be matched by the *Insider's* bid if the object is "good" and the *Insider* can afford it. In other words, the *Outsider's* maximization problem is

$$\max_{t \in [0,1]} u(t, q, \alpha, c) = \max_{t \in [0,1]} \{(1 - q)(-t - c) + qF(t)$$
$$\times ((1 + \alpha) - t - c) + q(1 - F(t))(-c)\},$$

or

$$\max_{t \in [0,1]} u(t, q, c, \alpha) = \max_{t \in [0,1]} \{-(1 - q)t$$
$$+ q\{F(t)((1 + \alpha) - t)\} - c\}, \tag{1}$$

where $c \geq 0$ is the cost of making a bid. Note that the *Insider* will match the *Outsider's* bid if the object is "good" with probability $q(1 - F(t))$ and the *Outsider's* payoff will be equal to $-c$. If the *Outsider* does not make a bid, her payoff is zero.

The optimal bid $\hat{t}$ must satisfy the *Outsider's* individual rationality constraint:

$$\hat{t} = \begin{cases} t^*, & \text{if } u(t^*, q, \alpha, c) > 0, \\ 0, & \text{if } u(t^*, q, \alpha, c) \leq 0, \end{cases} \tag{2}$$

where t^* is a solution of the maximization problem (1). We assume that if $t^* > 0$ and $u(t^*, q, c, \alpha) = 0$, the *Outsider* does not make a bid and has 0 payoff. We will write that the optimal bid is zero in this case, $\hat{t} = 0$. We will also assume that it is not optimal for the *Outsider* to win for sure by bidding $t = 1$:

$$u(1, q, \alpha, c) = q(1 + \alpha) - 1 - c < 0,$$

or

$$\alpha < \frac{1 + c - q}{q}.$$

3. Analysis

There are three key factors which make the Matching Auction work: (a) the imperfection in financial markets, (b) a high probability that the object is "good", and (c) a low cost of making a bid. We consider these factors in turn.

3.1. Imperfect financial markets

Financial markets are imperfect if borrowing money is a problem. The imperfection in financial markets is a decisive factor in overcoming the Winner's Curse problem in the Matching Auction. Suppose that the financial markets are perfect, or $F(t) = 0$ for any $t \in [0, 1]$. This means that the *Insider* not only *knows* the value of the object, but *can* also *match any bid* if he wants to. Then, the only optimal choice for the *Outsider* is to make a zero bid, $\hat{t} = 0$, because she can only win a "bad" object. This is a typical situation in takeover games. See, for example, Bazerman and Samuelson (1983) and Charness and Levin (2009).

Further, we will assume financial markets to be imperfect. Formally,

$$0 \leq F(t) \leq 1, \qquad F(0) = 0, \qquad F(1) = 1, \quad \text{and}$$
$$F'(t) = f(t) > 0 \quad \text{for } t \in [0, 1].$$

3.2. A "good" object

The first order condition for problem (1) is

$$-(1 - q) + qf(t)((1 + \alpha) - t) - qF(t) = 0, \tag{3}$$

where $f = F'$ is the density function.

The second order condition for problem (1) is

$$((1 + \alpha) - t)f'(t) - 2f(t) \leq 0. \tag{4}$$

The solution of Eq. (3), t^*, which satisfies (4), depends on the distribution function F, efficiency parameter α, and the probability that the object is "good", q. First, we show that t^* is an increasing function of q and α.

Proposition 1. $\frac{\partial t^*}{\partial q} \geq 0$ and $\frac{\partial t^*}{\partial \alpha} \geq 0$.

Proof. Denote

$$G(t^*, q, \alpha) = -(1 - q) + qf(t^*)((1 + \alpha) - t^*) - qF(t^*).$$

Then from Eq. (3), we can calculate $\frac{\partial t^*}{\partial q}$ and $\frac{\partial t^*}{\partial \alpha}$ as

$$\frac{\partial t^*}{\partial q} = -\frac{\frac{\partial G(t^*, q, \alpha)}{\partial q}}{\frac{\partial G(t^*, q, \alpha)}{\partial t^*}} = -\frac{1 + f(t^*)((1 + \alpha) - t^*) - F(t^*)}{q[f'(t^*)((1 + \alpha) - t^*) - 2f(t^*)]}$$

and

$$\frac{\partial t^*}{\partial \alpha} = -\frac{\frac{\partial G(t^*, q, \alpha)}{\partial \alpha}}{\frac{\partial G(t^*, q, \alpha)}{\partial t^*}} = -\frac{qf(t^*)}{q[f'(t^*)((1 + \alpha) - t^*) - 2f(t^*)]}.$$

From $t^* \in (0, 1)$, $F(t^*) \leq 1$ and the second order condition (4), we get that

$$\frac{\partial t^*}{\partial q} \geq 0 \quad \text{and} \quad \frac{\partial t^*}{\partial \alpha} \geq 0. \quad \square$$

Proposition 1 is intuitive: if the *Outsider* is more efficient, $\alpha_1 > \alpha_2$, or more confident that the object is "good", $q_1 > q_2$, then she is ready to bid more, $t_1^* > t_2^*$, in order to increase her winning chances. The following result is a corollary of this proposition.