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On the steady-state behavior of low-inertia power systems ⋆

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Abstract: Whereas conventional power systems heavily rely on bulk generation by synchronous machines, future power systems will be comprised of distributed generation based on renewable sources interfaced by power electronics. A direct consequence of retiring synchronous generators is the loss of rotational inertia, which thus far was the dominant time constant in a power system, as well as the loss of the generator controls, which are the main source of actuation of the power grid. Prompted by these paradigm shifts, we study the dynamic behavior of a nonlinear and first-principle low-inertia power system model including detailed power converter models and their interactions with the power grid. In this paper, we focus particularly on the admissible steady-state behavior of such a low-inertia power grid and derive necessary and sufficient control specifications for power converters.

Keywords: power system dynamics, steady-state behavior, port-Hamiltonian systems.

1. INTRODUCTION

The electric power system is currently undergoing a major transition towards integration of large shares of distributed generation connected by power electronic converters. Today power systems operation is based on bulk generation by synchronous machines and heavily relies on their rotational inertia for robustness. In contrast, future power systems will be based on renewable sources, distributed generation, and power electronics.

A direct consequence of retiring synchronous generators is the loss of rotational inertia, which thus far was the main reason for the grid's stability and robustness to disturbances. This results in larger, and more frequent, frequency deviations and jeopardizes the stability of the power grid (Tielens and Hertem, 2016; Winter et al., 2015). At the same time, analysis of such phenomena is a challenging problem because the power system physics are highly nonlinear, large-scale, and contain dynamics on multiple time scales from mechanical and electrical domains. As a result, analysis and control of conventional power system are typically based on reduced order models of various degrees of fidelity (Sauer and Pai, 1998).

A widely accepted reduced model of conventional power systems is a structure-preserving multi-machine model, where each generator model is reduced to the swing equation describing the interaction between the generator rotor and the grid, which is itself modeled at quasi-steady-state via the nonlinear algebraic power balance equations. While this prototypical model has proved itself useful (Sauer and Pai, 1998) its validity for conventional power systems has always been a subject of debate; see (Caliskan and Tabuada, 2015; Venezian and Weiss, 2016) for recent discussions. Because the derivation of this model crucially relies on time-scale separations induced by the rotational inertia of the generators its validity for low-inertia power systems is highly questionable. For instance, if power electronics are modeled as a constant power source and all generators are removed only algebraic equations remain. Moreover, it is not clear which dynamics of power electronics devices need to be included, or how the AC signals which connect the power electronics to the grid enter into the swing equation.

One approach to mitigate the loss of rotational inertia is to use power electronic devices to provide virtual inertia. These control schemes typically measure AC signals which are not present in the swing equation (Zhong and Weiss, 2011; D'Arco and Suul, 2013; Sinha et al., 2015). As a first step towards overcoming these limitations, we propose a first-principles model of a power system containing synchronous machines, DC/AC inverters, as well as transmission line and voltage bus dynamics. The models of the generator and the network are based on the detailed port-Hamiltonian power system model proposed in Fiaz et al. (2013) and combined with an averaged DC/AC inverter model proposed by Jouini et al. (2016). In contrast to typical inverter models, we explicitly consider the dynamics of the DC-link capacitor, which is the dominant time constant of the DC/AC inverter dynamics.

We seek answers to similar questions as in our previous works on conventional power systems (Arghir et al., 2016; Groß et al., 2016): under what conditions does the system admit a steady-state in which all three-phase AC signals are balanced, sinusoidal, and of the same synchronous frequency. We provide an algebraic characterization that specifies the state variables, control inputs, and a synchronous frequency such that the dynamics of the power system coincide with the desired steady-state dynamics.

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This analysis constructively leads to necessary conditions that any controller for the power system has to satisfy in steady-state. Specifically, we show that the set of states and control inputs on which the dynamics coincide is control-invariant if and only if the DC current supplied to the inverter as well as all generator inputs are constant, and the inverter switching block operates at the synchronous frequency. Several heuristic inverter control strategies ("heuristic" in the sense that they are not constructed from a priori specifications), such as droop control (Dörfler et al., 2016), virtual oscillator control (Sinha et al., 2015), synchronverters (Zhong and Weiss, 2011), generator emulation (D'Arco and Suul, 2013), matching control (Jouini et al., 2016), and grid-following control (Tabesh and Iravani, 2009), implicitly satisfy these specifications in steady-state. Moreover, the steady-state specifications for the generators justify assumptions used in the stability analysis of multi-machine networks (Caliskan and Tabuada, 2014).

2. NOTATION AND PROBLEM SETUP

2.1 Notation

We use $\mathbb{R}$ and $\mathbb{N}$, to denote the set of real numbers and integers, and e.g. $\mathbb{R}_{>0}$ to denote the set of positive real numbers. For column vectors $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ we use $(x, y) = [x^\top \ y^\top]^\top \in \mathbb{R}^{n+m}$ to denote a stacked vector, and for vectors or matrices x, y we use $\text{diag}(x, y) = \left[\begin{smallmatrix} x & 0 \\ 0 & y \end{smallmatrix}\right]$. Furthermore, I_n denotes the identity matrix of dimension n, $\otimes$ denotes the Kronecker product, and $\|x\| = \sqrt{x^\top x}$ denotes the Euclidean norm. Matrices of zeros and ones of dimension $n \times m$ are denoted by $\mathbb{0}_{n\times m}$ and $\mathbb{1}_{n\times m}$, and $\mathbb{1}_n$ denotes a column vector of ones of length n.

2.2 Dynamical Model of a Power Network

The power system model used in this work consists of n_g generators with index set $\mathbb{G} = \{1, \ldots, n_g\}$, n_i DC/AC inverters with index set $\mathbb{I} = \{n_g+1, \ldots, n_g+n_i\}$, n_v voltage buses with index set $\mathbb{V} = \{1, \ldots, n_v\}$, and n_t transmission lines with index set $\mathbb{T} = \{1, \ldots, n_t\}$. The AC voltage buses are partitioned into generator buses $\mathbb{V}_g = \{1, \ldots, n_g\}$, n_i inverter buses $\mathbb{V}_i = \{n_g+1, \ldots, n_g+n_i\}$, and n_l load buses $\mathbb{V}_l = \{n_g+n_i+1, \ldots, n_g+n_i+n_l\}$, i.e. $n_v = n_g+n_i+n_l$. The components of the power system as well as the main signals and parameters are depicted in Figure 1.

The model used in this manuscript combines a DC/AC inverter model proposed by Jouini et al. (2016) with a variant of the port-Hamiltonian power system model by Fiaz et al. (2013) used in Arghir et al. (2016). The reader is referred to these references for detailed derivations and discussions of the models. Based on the following assumption all three-phase AC signals are represented in (α, β) coordinates.

Assumption 1. All three-phase AC quantities are assumed to be balanced. Moreover, we assume that all three-phase electrical components (resistance, inductance, capacitance) have identical values for each phase.

Inverter Dynamics:
We consider a three-phase DC/AC inverter consisting of a DC-link capacitor, a switching block that modulates the DC-link capacitor voltage into an AC voltage, and an output filter. For the time scale of interest, we assume that the switching frequency is high enough and that the switching harmonics are suppressed by the output filter. By averaging the behavior of the switching block over one switching period, an averaged model of the inverter is obtained (Tabesh and Iravani, 2009; Jouini et al., 2016). The dynamics of the k-th inverter, with $k \in \mathbb{I}$, is given by:

$$\dot{q}_{I,k} = -G_{I,k} C_{I,k}^{-1} q_{I,k} + i_{sw,k}(\lambda_{I,k}, m_k) + i_{dc,k}, \tag{1a}$$

$$\dot{\lambda}_{I,k} = -R_{I,k} L_{I,k}^{-1} \lambda_{I,k} + C_k^{-1} q_k - v_{sw,k}(q_{I,k}, m_k), \tag{1b}$$

with DC-link capacitor charge $q_{I,k} \in \mathbb{R}_{\geq 0}$, output filter flux $\lambda_{I,k} = (\lambda_{I,\alpha,k}, \lambda_{I,\beta,k}) \in \mathbb{R}^2$. The AC-side capacitor, with charge $q_k = (q_{\alpha,k}, q_{\beta,k}) \in \mathbb{R}^2$, interconnects the inverter to the grid and will be described later in the model of the AC voltage bus dynamics. The control inputs of the inverter are the modulation signal $m_k \in \mathbb{R}^2$, which has to satisfy $\|m_k\| \leq 1$, and the current $i_{dc,k}$ supplied to the DC-link capacitor. In other words, we assume that $i_{dc,k}$ is supplied by a controllable source, e.g. a boost converter connected to photovoltaics and/or a battery.

The averaged output voltage $v_{sw,k}(q_{I,k}, m_k)$ and switching current $i_{sw,k}(\lambda_{I,k}, m_k)$ are given by:

$$i_{sw,k}(\lambda_{I,k}, m_k) = \tfrac{1}{2} \lambda_{I,k}^\top L_{I,k}^{-1} m_k, \tag{2a}$$

$$v_{sw,k}(q_{I,k}, m_k) = \tfrac{1}{2} C_{I,k}^{-1} q_{I,k} m_k \tag{2b}$$

The DC capacitance and conductance are denoted by $C_{I,k} \in \mathbb{R}_{>0}$ and $G_{I,k} \in \mathbb{R}_{>0}$ and $L_{i,k} = I_2 l_{I,k}$, $l_{I,k} \in \mathbb{R}_{>0}$ and $R_{I,k} = I_2 r_{I,k}$, $r_{I,k} \in \mathbb{R}_{>0}$ denote the inductance and resistance of the output filter.

Synchronous Generator Dynamics:
A generator with index $k \in \mathbb{G}$ is modeled by

$$\dot{\theta}_k = M_k{}^{-1} p_k \tag{3a}$$

$$\dot{p}_k = -D_k M_k{}^{-1} p_k - \tau_{e,k}(\lambda_k, \theta_k) + \tau_{m,k} \tag{3b}$$

$$\dot{\lambda}_k = -R_k \mathcal{L}_{\theta,k}^{-1} \lambda_k + \begin{bmatrix} C_k^{-1} q_k \\ v_{f,k} \end{bmatrix}, \tag{3c}$$

where $\lambda_k = (\lambda_{s,k}, \lambda_{f,k}) \in \mathbb{R}^3$ represents the stator flux linkage $\lambda_{s,k} = (\lambda_{\alpha,k}, \lambda_{\beta,k}) \in \mathbb{R}^2$ and rotor flux linkage $\lambda_{f,k} \in \mathbb{R}$, $p_k \in \mathbb{R}$ is the momentum of the rotor, and $\theta_k \in \mathbb{R}$ its angular displacement. The generator is actuated by the voltage $v_{f,k} \in \mathbb{R}$ across the excitation winding of the generator and the mechanical torque $\tau_{m,k} \in \mathbb{R}$ applied to the rotor. The electrical torque acting on the rotor is denoted by $\tau_{e,k}(\lambda_k, \theta_k) = \frac{\partial}{\partial \theta_k}(\frac{1}{2} \lambda_k^\top \mathcal{L}_{\theta,k}^{-1} \lambda_k)$. The inertia and damping of the rotor are given by $M_k \in \mathbb{R}_{>0}$ and $D_k \in \mathbb{R}_{>0}$, and the windings have resistance $R_k = \text{diag}(R_{s,k}, r_{f,k})$ with $R_{s,k} = I_2 r_{s,k}$, $r_{s,k} \in \mathbb{R}_{>0}$, and $r_{f,k} \in \mathbb{R}_{>0}$. The inductance matrix $\mathcal{L}_{\theta,k} : \mathbb{R} \to \mathbb{R}^{3\times 3}$ is a function of the angle θ_k and defined based on the stator inductance $L_{s,k} = I_2 l_{s,k} \in \mathbb{R}^{2\times 2}_{>0}$, mutual inductance $L_{m,k} = (l_{m,k}, 0) \in \mathbb{R}^2_{>0}$, and rotor inductance $l_{r,k} \in \mathbb{R}_{>0}$:

$$\mathcal{L}_{\theta,k} = \begin{bmatrix} L_{s,k} & \mathcal{R}_{\theta_k} L_{m,k} \\ L_{m,k}^\top \mathcal{R}_{\theta_k}^\top & l_{r,k} \end{bmatrix}, \ \mathcal{R}_{\theta_k} = \begin{bmatrix} \cos(\theta_k) & -\sin(\theta_k) \\ \sin(\theta_k) & \cos(\theta_k) \end{bmatrix}.$$

For notational convenience, we introduce skew symmetric matrices $j \in \mathbb{R}^{2\times 2}$ and $\mathcal{J} \in \mathbb{R}^{3\times 3}$:

$$j = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathcal{J} = \begin{bmatrix} j & \mathbb{0}_{2\times 1} \\ \mathbb{0}_{1\times 2} & 0 \end{bmatrix}.$$