Networks of open systems
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Abstract

Many systems of interest in science and engineering are made up of interacting subsystems. These subsystems, in turn, could be made up of collections of smaller interacting subsystems and so on. In a series of papers David Spivak with collaborators formalized these kinds of structures (systems of systems) as algebras over presentable colored operads (Spivak, 2013; Rupel and Spivak, 2013; Vagner et al., 2015).

It is also very useful to consider maps between dynamical systems. This is the point of view taken by De Ville and Lerman in the study of dynamics on networks (De Ville and Lerman, 2015 [4,5]; De Ville and Lerman, 2010). The work of De Ville and Lerman was inspired by the coupled cell networks of Golubitsky, Stewart and their collaborators (Stewart et al., 2003; Golubitsky et al., 2005; Golubitsky and Stewart, 2006).

The goal of this paper is to describe an algebraic structure that encompasses both approaches to systems of systems. More specifically we define a double category of open systems and construct a functor from this double category to the double category of vector spaces, linear maps and linear relations. This allows us, on one hand, to build new open systems out of collections of smaller open subsystems and on the other to keep track of maps between open systems. Consequently we obtain synchrony results for open systems which generalize the synchrony results of Golubitsky, Stewart and their collaborators for groupoid invariant vector fields on coupled cell networks.

1. Introduction

Many systems of interest in science and engineering are made up of interacting subsystems. These subsystems, in turn, could be made up of collections of smaller interacting subsystems and so on. These kind of structures have been formalized by David Spivak with collaborators as algebras over presentable colored operads [1–3]. There are several variants of these operads; they depend on the kinds of systems one is interested in. Since the subsystems are supposed to receive input from other subsystems they are conveniently modeled as open (a.k.a. control) systems; we review open systems in Section 2. Informally an open system is a dynamical system that receive inputs from other systems. There are several formal models of open systems starting with collections of vector fields that depend on parameters. In [3] the input–state–output model is used.

One of the fundamental problems in the theory of (closed) dynamical systems is finding or, failing that, proving the existence of equilibria, periodic orbits and, more generally, other invariant submanifolds. This amounts to finding/proving the existence of maps between dynamical systems. For example, a map from a point to our favorite closed system is an equilibrium, maps from circles are periodic orbits, and so on. Thus it is highly desirable to have a systematic way of constructing maps between dynamical systems.

One can view a network as a pattern of interconnection of open system. In [3] a network is formalized as a morphism in the colored operad of wiring diagram — the morphism encodes the pattern. In the work of De Ville and Lerman [4–6], which was...
Inspired by the coupled cell networks of Golubitsky, Stewart and their collaborators [7–9], networks are encoded by directed graphs. In contrast to [3] networks in [4–6] are viewed as objects in a category, and the main result is a good notion of a map between networks. The notion leads to a combinatorial recipe for a construction of maps of closed dynamical systems out of appropriate maps of graphs. We will show in this paper that the networks of [4–6] can be viewed as particular kinds of morphisms in a colored operad (Proposition 10.3). The morphisms of networks of [4–6], on the other hand, have no obvious interpretation in the operadic language.

In this paper we generalize both approaches (directed graphs and operadic) to networks of open systems. This allows us, on one hand, to build new large open systems out of collections of smaller open subsystems and on the other hand keep track of maps between open systems. Consequently we obtain synchrony results for open systems which generalize the synchrony results of Golubitsky, Stewart and their collaborators for groupoid invariant vector fields on coupled cell networks (see for example [7–9]).

Networks of open systems as such are not new. For example, networks of open systems are implicit in the work of Field [10]. They are also implicit in the work of Golubitsky, Pivato, Torok and Stewart [7, 8] and their collaborators. Special cases of networks of open systems present in the coupled cell network formalism were made explicit in [4–6]. Maps between open systems are not new either. For example the category of open systems has been explicitly introduced by Tabuada and Pappas [11].

What is new in this paper is a general notion of maps between networks (Definition 9.1) and a dynamical/control system interpretation of these maps (Theorem 9.5). We frame this notion in terms of double categories. In particular the results of this paper subsume and extend the results of [4–6], as we explain in Section 10.

The paper assumes that the reader is comfortable with viewing continuous time dynamical systems as vector fields on manifolds. By necessity the paper also uses a certain amount of category theory, which we try to keep down to a minimum. We expect the reader to be comfortable with the universal properties of products and coproducts and have a nodding acquaintance with 2-categories, but not much more than that. Some of the results of the paper are expressed in the language of symmetric monoidal categories and the corresponding colored operads. A reader who may be unfamiliar with monoidal categories may safely skip the corresponding parts of the paper. We also use the language of (strict) double categories. Since double categories are somewhat less common, we do not expect any familiarity with them on the part of the reader. Strict double categories are reviewed in Section 8.

Organization of the paper. We remind the reader that the noun phrases “open system” and “control system” are used interchangeably throughout the paper.

The paper starts by recalling a definition of an open/control system (Definition 2.3) and reviewing the category of open systems of Tabuada and Pappas [11]. We then construct a symmetric monoidal category $\text{SSub}^{\text{int}}\text{op}$ whose objects are surjective submersions. In a coordinate-free approach to control theory a surjective submersion $a$ gives rise to a vector space $\text{Crl}(a)$ of control systems. We extend the assignment $a \mapsto \text{Crl}(a)$

to a morphism of symmetric monoidal categories

$$\text{Crl} : (\text{SSub}^{\text{int}}\text{op}) \to \text{Vect},$$

where Vect is the category of real vector spaces and linear maps with the monoidal product being given by direct sum $\oplus$.

Recall that a symmetric monoidal category $A$ defines a colored operad $\mathcal{O}(A)$. We interpret a morphism in the operad $\mathcal{O}(\text{SSub}^{\text{int}}\text{op})$ as a pattern of interconnection of open systems and think of it as a network of open systems. The monoidal functor $\text{Crl}$ turns the colored operad $\mathcal{O}(\text{Vect}, \oplus)$ into an algebra over the operad $\mathcal{O}(\text{SSub}^{\text{int}}\text{op})$ (Section 4).

In Section 5 we review the category of lists $\text{FinSet}/\mathcal{C}$ in a category $\mathcal{C}$. The objects of $\text{FinSet}/\mathcal{C}$ are finite unordered lists of objects of $\mathcal{C}$. This is done to facilitate the comparison of the operad $\mathcal{O}(\text{SSub}^{\text{int}}\text{op})$ with the operad of wiring diagrams of [3]. There are also other reasons for introducing the categories of lists that will become apparent later. We then revisit the algebra $\mathcal{O}\text{Crl} : \mathcal{O}(\text{SSub}^{\text{int}}\text{op}) \to \mathcal{O}\text{Vect}$ introduced earlier in Section 5.2.

We carry out the comparison of the operad $\mathcal{O}(\text{SSub}^{\text{int}}\text{op})$ with the operad of wiring diagrams in Section 6. The main difference between the two operads and their respective algebras is philosophical. Namely, the approach of [3] is to treat an open system as a black box—the space of internal states is completely unknown while the algebra supplies all possible choices of internal state spaces. By contrast in this paper we treat the space of internal states (and the total space) as known and have the algebra supply the possible choices of dynamics on a given total space.

The next two sections are technical. The main result of Section 7 is Lemma 7.1. This lemma, in effect, is half of the proof of the main theorem of the paper, Theorem 9.5. The results of Section 7 are used to motivate the introduction of double categories, which is carried out in Section 8. The two main results of Section 8 are Lemma 8.8 (which is a reformulation of Lemma 7.1 in terms of double categories) and Lemma 8.12.

Finally in Section 9 we introduce our notion of maps between networks (Definition 9.1) and interpret it in terms of maps of open systems (Theorem 9.5). In Section 10 we show that the networks of [4] (hence the coupled cell networks of Golubitsky, Stewart et al.) are a special case of the networks in the sense of Definition 3.9. We then show that Theorem 3 of [4] (which is the main result of that paper) is an easy consequence of Theorem 9.5.
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