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# Energy-efficient shortest routes for electric and hybrid vehicles

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ABSTRACT

Electric and hybrid vehicles are a big step towards a greener mobility, but they also open up completely new questions regarding the shortest path problem and the planning of trips. Since recharging an electric car will take much longer than refilling conventional fossil fuels, we have to balance between speed and range and we have to choose stops for charging wisely. For hybrid vehicles, a symbiosis between navigation system and power train control to choose a path with optimal phases for depleting and recharging the battery may yield much more energy-efficient paths. In this paper, we develop an appropriate model for finding shortest routes for these kinds of vehicles, which is mainly a constrained shortest path problem with convertible resources and charging stations. We study properties of solutions by classifying several types of cycles that may occur in the optimal route. We state sufficient conditions to exclude some of these cycle classes and we derive appropriate approximation schemes with provable quality and strict feasibility. We also study the related network flow problem for operating fleets of electric vehicles, e.g., shared vehicles or buses in urban areas.



## 1. Introduction

Electric powered vehicles can be the backbone of environmental-friendly individual traffic. Using green electricity, we have the chance to overcome more than 200 years of burning fossil fuels since the development of the first steam locomotives and steamboats.

However, at least today the main disadvantage of electric vehicles is a relatively short range compared to cars with internal combustion engines. Hence, charging stations have to be visited more often, but they are far less common than gas stations. Moreover, charging requires more time than refilling gasoline.

Just to provide some numbers: there are about 115,000 gas stations in the United States according to the U.S. Census Bureau. The number of Tesla Superchargers, where charging only takes about 30 min, is growing quickly, but at the beginning of 2016, there are only about 250 Supercharger locations in the United States. Of course, there are much more standard charging locations, but these chargers usually provide only 30–40 Ampere (7-10 kW). There, a full recharge of the 85 kWh battery of a Tesla Model S takes 10–12 h. Considering the 3.5 million square miles land area of the United States, the next

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gas station is only 2.2 miles away in average, whereas the next Supercharger is 76 miles away in average assuming a uniform distribution.

But we should not only consider long distance trips. Especially in urban traffic with a lot of accelerating and braking, electric vehicles are very energy efficient and they make an important contribution to air pollution control. Here, the typical car user uses the vehicle for several short trips within one day, interrupted by several activities. For example, a day starts with taking the kids to school, going to work, going to some leisure activity in the afternoon, going to the supermarket and returning home in the evening. A smart navigation device should plan consecutive routes at once and it should plan the recharging of the battery such that it coincides with the activities. Maybe there is a standard plug at work or a fast charging station at the supermarket, hence, the battery level can be kept high without spending unused times waiting next to the car while charging.

Besides electric vehicles, hybrid vehicles are in-between classical combustion engine driven cars and electric vehicles. They are equipped with both internal combustion engine and electrical motor. Here, the electric drive provides a better efficiency in many situations, e.g., for accelerating in city traffic. The combustion engine is mainly used as a range extender. Plug-in hybrids can also be charged via an external connection for power supply.

For both kinds of vehicles, electric and hybrid, the electrical drive can also be used as a generator. This allows recuperation during downhill driving or braking. In hybrid cars, one may also use the combustion engine to drive the generator. In other words, we can convert additional gasoline into electrical energy, store this energy in the battery, and use it later.

Consequently, many new navigational questions arise. Instead of asking for a shortest path by means of time or distance, we can ask for the route with the least energy consumption. If our electric vehicle does not provide enough range, we have to decide which charging stations should be approached. The on-board navigation system can recommend to slow down if this saves one stop for charging and therefore time. This is even more interesting when we consider different charging stations, e.g., standard plug sockets, fast charging stations or even battery swaps. Furthermore, we do not have to charge the battery completely, which could be a waste of time, since the charge rate typically decreases at higher charge levels.

For hybrid vehicles, the full potential will only be realized if navigation and engine control exchange data. For example, it is obviously a bad idea to arrive on top of a mountain with a completely charged battery. Recuperation on the subsequent downhill road yields energy for free that could not be stored in a fully charged battery. Hence, engine control will benefit from route data. On the other hand, more energy efficient routes can be found if the navigation system anticipates the behavior of the power train. Routing now involves a much more sophisticated planning. Besides the route itself, we have to determine which kind of drive should be used on each road segment.

Electric powered vehicles are also interesting for many applications where a fleet of vehicles is operated. For example, several companies providing car sharing or similar mobility services are planning to migrate to electric vehicles. More and more automated guided vehicles (AGV) with battery-electric drive units are used in container terminals around the world. Moreover, battery powered buses are used in public transport. Operating such a fleet efficiently requires operating suitable charging stations[1]. However, since such a charging station has a limited capacity itself, planning is needed to charge all these batteries in time. This directly leads to the corresponding network flow problem of routing with limited charging capacity.

*Our contribution.* In this paper, we introduce a general model for the routing of electric and hybrid vehicles with convertible resources and intermediate stops at charging stations. After defining the problem in Section 3, an analysis of the structure of optimal solutions in Section 4 reveals, that a shortest route may contain cycles. Yet, we formulate constraints on resource consumption and cost functions to exclude the pathological cases. Equipped with these cycle prevention conditions, we study algorithms for finding such fast and energy-efficient routes in Section 5 and we present an appropriate approximation scheme. Here, we restrict ourselves to strict feasibility, i.e., the resource consumption of the battery is not subject to any rounding. Only costs (time or fossil fuel consumption, respectively) are affected by rounding and the acceptable total error can be chosen in advance. Furthermore, if there exists a feasible path, this path has to be found. In particular, the trade-off between speed and range is addressed in Section 5.1. Subsequently, Section 5.2 presents an algorithm for optimal recharging. We also give a less precise approach in Section 5.3 that is more suitable for practical purposes. Finally, we discuss the consequences of cycles for the corresponding flow problem in Section 7.

Please note that some of the results were already presented at conferences and published in the proceedings or as a preprint (Merting et al., 2015; Schwan and Strehler, 2016), partially without proofs. Here, we give a complete presentation with many additional results, proofs, and examples as well as an improved, simpler, and unified version of the approximation scheme.

## 2. Related literature

Both problems are related to the ConstrainedShortestPath problem (CSP). In this problem, we are given a graph $G = (V, E)$ with a cost function $c : E \to \mathbb{R}$ and lengths or resources $r_i : E \to \mathbb{R}$. In general, costs and resources are assumed to be non-negative. Now, one seeks for a shortest path $P$ from a vertex $s$ to a vertex $t$ which obeys the resource constraints $R_i$, $i \in N = \{1, \ldots, k\}$. That is, we want to find an $s-t$-path $P$ minimizing $\Sigma_{e \in P} c(e)$ such that $\Sigma_{e \in P} r_i(e) \leq R_i \, \forall i \in N$.

[1] Battery swapping is the most common choice, since the vehicle can directly return to its purpose with a new battery and the old battery is charged outside the vehicle.