Journal of Economic Behavior & Organization 76 (2010) 615–634

Contents lists available at ScienceDirect

Journal of Economic Behavior & Organization

journal homepage: www.elsevier.com/locate/jebo

Dynamics of moving average rules in a continuous-time financial market model☆

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ARTICLE INFO

Article history:
Received 23 April 2009
Received in revised form 13 August 2010
Accepted 16 August 2010
Available online 22 September 2010

JEL classification:
C62
D53
D84
G12

Keywords:
Asset price
Financial market behavior
Heterogeneous beliefs
Stochastic delay differential equations
Stability
Bifurcations
Stylized facts

ABSTRACT

Within a continuous-time framework, this paper proposes a stochastic heterogeneous agent model (HAM) of financial markets with time delays to unify various moving average rules used in discrete-time HAMs. The time delay represents a memory length of a moving average rule in discrete-time HAMs. Intuitive conditions for the stability of the fundamental price of the deterministic model in terms of agents' behavior parameters and memory length are obtained. It is found that an increase in memory length not only can destabilize the market price, resulting in oscillatory market price characterized by a Hopf bifurcation, but also can stabilize an otherwise unstable market price, leading to stability switching as the memory length increases. Numerical simulations show that the stochastic model is able to characterize long deviations of the market price from its fundamental price and excess volatility and generate most of the stylized facts observed in financial markets.

1. Introduction

Despite the efficient market hypothesis of financial markets in the academic finance literature (see Fama, 1970), the use of technical trading rules, such as moving average rules, still seems to be widespread amongst financial market practitioners (see Allen and Taylor, 1990; Taylor and Allen, 1992). Technical analysts or "chartists", who use various technical trading rules, attempt to forecast future prices by the study of patterns of past prices and other summary statistics about security trading. Basically, they believe that shifts in supply and demand can be detected in charts of market movements.

Earlier empirical literature on stock returns finds evidences that daily, weekly and monthly returns are predictable from past returns. Pesaran and Timmermann (1994, 1995) present evidence on the predictability of excess returns on common stocks for the S&P 500 and Dow Jones Industrial portfolios, and examine the robustness of the evidence on the predictability

☆ Financial support from the Australian Research Council (ARC) under a Discovery Grant (DP0773776) and the University of Technology, Sydney under an ECRG (Zheng) and a research grant from the Faculty of Business (He) is gratefully acknowledged. The authors would like to thank two anonymous reviewers for providing helpful comments.

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0167-2681/$ – see front matter.
doi:10.1016/j.jebo.2010.08.005

of U.S. stock returns. There have been various studies of the profitability of technical analysis; see Frankel and Froot (1986, 1990), Brock et al. (1992), Neely et al. (1997), Gençay (1998) and Fernandez-Rodriguez et al. (2000).

Most of the cited research has focused on empirical studies. Recent studies, such as Lo et al. (2000),Boswijk et al. (2000) and Goldbaum (2003), have also examined explicitly the profitability of technical trading rules and the implications for market efficiency. Over the last two decades, various heterogeneous agent models (HAMs) have been developed to explain a range of market behavior. By incorporating bounded rationality and heterogeneity, HAMs have successfully explained many types of features (such as market booms and crashes, long deviations of the market price from the fundamental price), the stylized facts (such as skewness, kurtosis, volatility clustering and fat tails of returns), and various power laws (such as the long memory in return volatility) observed in financial markets. We refer the reader to Hommes (2006), LeBaron (2006) and Chiarella et al. (2009) for surveys of the recent developments in this literature.

To examine the role of moving average rules in market stability theoretically, Chiarella et al. (2006a) recently propose a discrete-time HAM in which demand for traded assets has both a fundamentalist and a chartist components. The chartist demand is governed by the difference between the current price and a moving average (MA). They show analytically and numerically that the MA plays a complicated role on the stability of financial markets. In particular, when the activities of the market participants (such as the fundamentalist and the trend follower) are balanced in certain way, an increase in the memory length used in the MA can stabilize the market; otherwise, it is a source of market instability, and the interaction of the MA and market noise can lead to the tendency for the market price to take long excursions (associated with long memory length) away from the fundamental price. We also refer to Levy et al. (2000), Chiarella and He (2001), Zschischang and Lux (2001) and Anufriev and Dindo (2009) for the related discussion on the standard MA and to Chiarella et al. (2006b) for weighted MA in the discrete time HAMs.

Interestingly enough, most of the HAMs in the literature are in discrete-time rather than continuous-time setup. The discrete-time setup facilitates economic understanding and mathematical analysis, but it also faces some limitations when expectations of agents are formed in historical prices over different time periods. In particular, when dealing with MA rules, different lag lengths used in the MA rules lead to different dimensions of the system which need to be dealt with differently; see Chiarella et al. (2006a). Very often, a theoretical analysis on the impact of the memory length used in MA is difficult when the dimension of the system is high, which is the case as the memory length used in the MA becomes large. To overcome this difficulty, this paper proposes a heterogeneous agent model of financial markets in a continuous-time framework with time delay, which represents a memory length of a moving average rule in discrete-time HAMs, to study the impact of the memory length. The financial market consists of a group of fundamentalists and a group of trend followers using a weighted average of historical prices as price trend. The trend followers are assumed to react to buy-sell signals generated by the difference between the current price and the price trend which is formed as an integral with a distributed delay, representing a MA of the historical prices with an exponential decaying weight over a memory length. By incorporating random fundamental price and noisy demand from noise traders in the market, the model is described mathematically by a system of stochastic differential equations with time delay. The continuous-time model provides a uniform treatment on various MA rules used in the discrete-time model.

Development of deterministic delay differential equation models to characterize fluctuation of commodity prices and cyclic economic behavior has a long history; see, for example, Haldane (1932), Kalecki (1935), Goodwin (1951), Larson (1964), Howroyd and Russell (1984) and Mackey (1989). The development further leads to the studies on the effect of policy lag on macroeconomic stability; see, for example, Phillips (1954, 1957), Asada and Yoshida (2001) and Yoshida and Asada (2007). Recently, by introducing noise processes into a simple price model with a delay dependent growth rate, Küchler and Platen (2007) extend the study to examine the joint effect of time delay and randomness. Though there is a growing study on various market behaviors and, in particular, the stylized facts, volatility clustering and long memory observed for high frequency (such as daily) returns in the discrete-time HAMs (see Lux, 2009, in press, for recent surveys), in our knowledge, using stochastic delay differential equations to study these features in financial markets is relatively new. This paper serves as the first step to extend the current HAMs from discrete-time to continuous-time within a time delay framework.

By developing a stochastic HAM in continuous time with delays, this paper focuses on the impact of the behavior of heterogeneous agents and, in particular, the role of the memory length in market stability. For the underlying deterministic delay differential equation model, the stability of the fundamental price in terms of agents' behavior parameters and time delay are analyzed. Consistent with the results obtained in the discrete-time model in Chiarella et al. (2006a), it is found that an increase in the memory length has a double edged effect on the stability, meaning that time delay can either destabilize or stabilize the market price. However, different from the discrete-time model, it is also found that, depending on the behavior of the fundamentalists and trend followers, an increase in the memory length can lead to stability switching of the market price, which is new in the HAMs.[1] For the corresponding stochastic model, we demonstrate that the model is able to generate many market phenomena, such as market bubbles, crashes and long deviations of the market price from the fundamental price, and most of the stylized facts, including non-normality, volatility clustering, and power-law behavior of high-frequency returns, observed in financial markets.

[1] It should be stressed that the double edged role on the stability of time delay is not new in applied mathematical literature, in particular in mathematical biology literature; see for example MacDonald (1978) and Beretta et al. (1988).