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Untangling complex networks: Risk minimization in financial markets through accessible spin glass ground states

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ABSTRACT

Recurrent international financial crises inflict significant damage to societies and stress the need for mechanisms or strategies to control risk and tamper market uncertainties. Unfortunately, the complex network of market interactions often confounds rational approaches to optimize financial risks. Here we show that investors can overcome this complexity and globally minimize risk in portfolio models for any given expected return, provided the margin requirement remains below a critical, empirically measurable value. In practice, for markets with centrally regulated margin requirements, a rational stabilization strategy would be keeping margins small enough. This result follows from ground states of the random field spin glass Ising model that can be calculated exactly through convex optimization when relative spin coupling is limited by the norm of the network's Laplacian matrix. In that regime, this novel approach is robust to noise in empirical data and may be also broadly relevant to complex networks with frustrated interactions that are studied throughout scientific fields.

1. Introduction

Large and abnormal fluctuations in financial markets can spread to other parts of the global economy with untoward and often incalculable effects. Therefore, a key priority is to minimize risks and contain their propagation in spite of the tendencies of current financial markets to the contrary [1,2]. Important examples of such market places include exchanges where stocks, commodities, futures and other financial products can be bought and sold short by using leverage on margin accounts held by investors. A central financial decision problem in these markets is, for a given expected return r_P, to distribute the available capital among multiple assets, which comprise a portfolio P of size n, so as to minimize the overall risk.

In portfolio selection models this goal can be mathematically formulated as finding the global minimum of a risk function [3–6], $R = 1/2\sum_{i,k=1}^{n} C_{ik}p_ip_k - \sum_{i=1}^{n} p_ir_i - \gamma\sum_{i=1}^{n} p_is_i$, where p_i is the positive or negative amount of capital invested in asset i, and $s_i = \text{sign}(p_i) \in \{-1, 1\}$ are binary *spin* variables; r_i is the expected return of asset i such that $r_P = \sum_{i=1}^{n} r_ip_i$; C_{ik} is the covariance between assets i and k; and γ is the margin account requirement which sets the fraction of capital that the investor must deposit in a margin account before buying or selling short assets. With the inverse C^{-1} of the covariance matrix C the minimum risk distribution $p = (p_1, \ldots, p_n)$ becomes $p_i = \sum_{k=1}^{n} C_{ik}^{-1}r_k + \gamma\sum_{k=1}^{n} C_{ik}^{-1}s_k$. It is known that finding the absolute risk minimum is computationally equivalent to the ground state problem of the random field Ising model [3,5]. This is evident after inserting p into the risk function while neglecting fixed terms that do not depend on spin variables which gives $R = -1/2\sum_{i,k=1}^{n} J_{ik}s_is_k - \sum_{i=1}^{n} h_is_i$, and where we introduced an interaction term $J_{ik} = \gamma C_{ik}^{-1}$ and a random local field $h = (h_1, \ldots, h_n)$ with $h_i = \sum_{k=1}^{n} C_{ik}^{-1}r_k$. Covariance among assets can be both positive and negative

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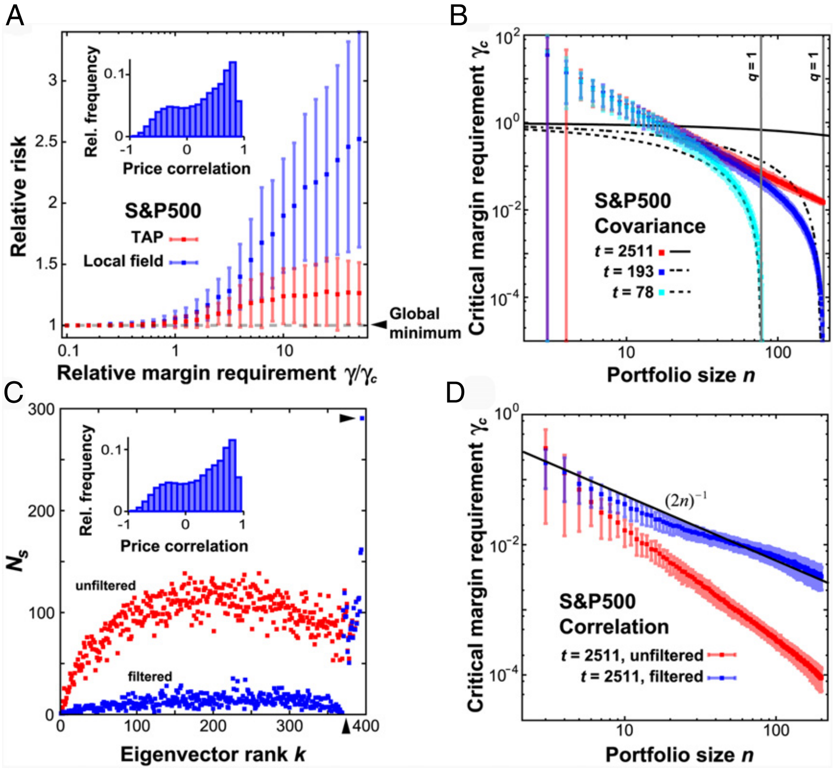

Fig. 1. (A) Portfolio risk can be globally and rationally minimized if the relative margin requirement satisfies $\gamma/\gamma_c < 1$. In contrast, for $\gamma/\gamma_c > 1$ the estimated risk undergoes large fluctuations above the optimum. Red data points ("TAP") give the risk from solutions of the TAP equation for $n = 16$ with randomly selected assets from the S&P500 price data, and with a random field $h = (h_1, \ldots, h_n)$ with $|h_i| \leq 1$. Blue data points ("Local field") depict the risk obtained by taking the sign of local field h. Error bars represent standard deviations after 128 random trials. Inset shows the distribution of price correlations between all pairs in the $m = 395$ assets taken from the S&P500 index. (B) Estimated critical margin requirement as a function of portfolio size $n \leq m$ and for three different choices of price samples, $t = \{2511, 193, 78\}$, where stock prices were selected every $\{1, 13, 32\}$ days, respectively. Error bars represent standard deviations from 128 random selections in the S&P500 price data. Black solid and dashed graphs represent the function $\left(1 - \sqrt{n/t}\right)^2$. (C) The inverse partition ratio $N_s = \sum_{l=1}^{m} (u_l^k)^4$ for each normalized eigenvector u^k of the $m \times m$ correlation matrix $\tilde{C}$ ranked by its increasing eigenvalues [11]. Red dots represent the unfiltered correlation matrix which, up to a rank of $k = 372$, follow a semicircle distribution; blue dots represent the filtered correlation matrix after setting all eigenvalues with lower rank to zero, i.e. those in size smaller than $\lambda_{\max}$. Inset shows the resulting histogram of pairwise price correlations after filtering. (D) Estimated critical margin requirement γ_c from the S&P500 correlation matrix $\tilde{C}$ before (red) and after (blue) eigenvalue filtering. Black solid line represents the graph $(2n)^{-1}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(see, for example, inset in Fig. 1(A)), and globally minimizing risk means finding a ground state of the random field Ising model with random spin glass interactions, which in general belongs to the class of NP-complete decision problems [7,8] and for which efficient algorithms are not known. This computational intractability arises from the non-convexity of the cost function R; non-convex problems are much harder to solve computationally than convex optimization problems for which efficient algorithms do exist [9]. In the context of financial markets, the non-convexity of the spin glass model prevents equilibration into an optimum ground state and is viewed as an inherent source of risk [10,3].

2. Accessible ground states in the spin glass Ising model with random field

Here we demonstrate that ground states are efficiently accessible in the random field spin glass Ising model provided the margin requirement γ remains below a critical value, which we define as $\gamma_c = \|L\|^{-1} = \left[\max_i \left(\sum_{k=1}^{n} |L_{ik}|\right)\right]^{-1}$, where $L = D - C^{-1}$ is the network's Laplacian matrix, with $D = \operatorname{diag}\left(\sum_{k=1}^{n} C_{ik}^{-1}\right)$ and $\|L\|$ is its maximum norm. This upper bound on the margin requirement ensures that there exists a related but convex risk function $R_c = 1/2 \sum_{i,k=1}^{n} J_{ik}(s_i - s_k)^2 + \sum_{i=1}^{n} (h_i - s_i)^2$, which in matrix form reads $R_c = (s - h)^{\mathrm{T}}(s - h) + \gamma s^{\mathrm{T}} L s$. We note that in the special and simpler case with non-negative interactions $J_{ik} \geq 0$ similar objective functions have been studied in semi-supervised machine learning [12]. In the more challenging spin glass case, our prerequisite $\gamma < \gamma_c$ makes the Hessian matrix $H_c = 1 + \gamma L$ positive definite such that R_c remains convex with one global minimum even if the interaction is described by a random mix of positive and negative numbers. Let s denote the minimum configuration in R_c obtained after convex optimization, then s also depicts the ground state s^* of the spin glass Ising model with a random field because assuming the contrary, $R(s) > R(s^*)$, leads to a