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Operational modal identification in the presence of harmonic excitation

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ABSTRACT

The dynamic behavior of structures can be studied by the identification of their modal parameters. Classical modal analysis methods are based on the relation between the forces applied to structures (inputs) and their vibration responses (outputs). In real operational conditions it is difficult, or even impossible, to measure the excitation. For this reason, operational modal analysis approaches which consider only output data are proposed. However, most of these output-only techniques are proposed under the assumption of white noise excitation. If additional components, like harmonics for instance, are present in the exciting force, they will not be separated from the natural frequencies. Consequently, this assumption is no longer valid. In this context, an operational modal identification technique is proposed in order to only identify real poles and eliminate spurious ones. It is a method based on transmissibility functions.

The objective of the proposed paper is to identify modal parameters in operational conditions in the presence of harmonic excitations. Identification is performed using a method based on transmissibility measurements and then with the classical stochastic subspace identification method, which is based on white noise excitation. These two methods are first applied to numerical examples and then to a laboratory test. Results validate the novel ability of the method based on transmissibility measurements to eliminate harmonics, contrary to the stochastic subspace identification approach.

1. Introduction

Modal analysis [1–3] is used to identify mode shapes, natural frequencies and damping ratios under vibrational excitation. These methods are efficient tool for detecting damage in structures, controlling them, and determining their structural stability. Modal parameters are initially identified using experimental modal analysis (EMA) [4]. This technique exploits the Frequency Response Function (FRF) of the structure, which represents the relation between the excitation and the vibrational response of the structure. Resonance frequencies appear as peaks in the measured frequency response functions. From these FRFs, modal parameters are identified using various curve-fitting techniques. Many excitation forms and experimental setups exist, their choice depending on the structure's complexity. Usually, the EMA is carried out under impact hammer and/or shaker excitation. The major drawback to experimental modal tests is that both artificially applied forces and resulting structural vibration responses need to be measured. In practice, the measurement of the exciting force is not always possible. In EMA, tests are performed at rest, and the dynamic properties of structures at rest vary from those of structures in operational conditions [5], which can significantly influence the identified modal model.

For these reasons, modal identification techniques are developed and operational modal analysis (OMA) is proposed [6,7], where the modal properties are estimated from responses only. Various operational modal identification techniques are proposed, example the Stochastic Subspace Identification approach (SSI) [8,9]. However, OMA methods have limitations when applied to practical cases. One limiting constraint of OMA is that the non-measured excitation of the system in operation must be a stochastic realization (white noise) [6]. This implies that if harmonic components are present in addition to random excitation, standard OMA procedures cannot be applied in a straightforward way. Harmonic components are sometimes considered as virtual modes in the identification, but when the harmonic excitation frequencies are close to eigenfrequencies, the standard OMA approaches may break down [10].

Several indicators for the separation of structural and harmonic modes in output-only modal identification are proposed. One of the most widely-used methods is based on the Probability Density Function

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(PDF) of harmonic and structural responses. The PDF of a structural response is a distribution with only one peak, and the PDF of a harmonic response is a distribution with two peaks. This difference was originally illustrated by Lago [11]. Kurtosis criteria have also been used to identify harmonic components and structural modes, [12–16]. Kurtosis is defined as the fourth central moment of the PDF, normalized with respect to the standard deviation. In addition to the above-mentioned methods, knowledge of the damping ratios is an a priori indicator to distinguish between harmonics and structural poles. Generally, the damping ratios of real poles vary between 0.1% and 2%. This information enables modes with negative and high damping to be eliminated [7]. The Modal Assurance Criterion (MAC) [17] is also an effective tool to distinguish a structural mode from a harmonic one. The MAC value between a structural mode shape and a mode shape corresponding to a harmonic component will show a low correlation. Specific numerical filters have also been developed [18] in order to eliminate harmonic components from the measured response. However, in practice filters are not perfect, and if the harmonic frequency is close to resonant frequencies, the filtering will disturb the response so that the identified modal parameters are perturbed.

In order to overcome the white noise excitation assumption and consequently to identify modal parameters in the presence of harmonic excitation, an operational modal identification method is proposed by Devriendt et al. [10,19–21]. This method is based on transmissibility measurements. The significant advantage of this approach is its independence from the nature of the excitation. Consequently, the presence of harmonics will not disturb the identified modal model.

The main objective of this paper is to identify the modal parameters of structures in the presence of harmonic components using the Transmissibility Function-Based method (TFB). The decision concerning whether a particular mode is structural or not is based on a singular value decomposition of the system's transmissibility matrix. The identified eigen-parameters are then compared to those obtained using the classical SSI method, which is based on the white noise excitation assumption. The idea behind this comparison is to demonstrate that the TFB method is a particular OMA which can eliminate harmonics and identify only the real poles of the structure. The paper is organized as follows. In Section 2, a brief description of the TFB and SSI techniques is provided. In Section 3, the OMA technique based on transmissibility measurements is applied to a numerical model and to a cantilever beam test. Results are then compared with those obtained via the SSI technique. Finally, Section 4 concludes the paper.

2. Operational modal identification techniques

2.1. Modal identification method based on transmissibility functions (TFB)

OMA approaches are generally based on the assumption of white noise processes for operational excitations. However, this assumption is hard to respect in real situations [10,22]. The method based on transmissibility functions (TFB) is independent from the nature of the excitation, and solves the problem of the presence of harmonic components. The use of transmissibility functions was proposed by Devriendt et al. [19] as a new approach in operational modal analysis. The expression of the frequency response at a point i under an excitation at a point k is written as:

$$X_i^k(s) = H_{ik}(s)F_k(s) \tag{1}$$

A transmissibility function is defined as the ratio between the motion response $X_i^k(s)$ and the reference motion response $X_j^k(s)$ under a single force located at k.

$$T_{ij}^k(s) = \frac{X_i^k(s)}{X_j^k(s)} = \frac{H_{ik}(s)F_k(s)}{H_{jk}(s)F_k(s)} = \frac{H_{ik}(s)}{H_{jk}(s)} \tag{2}$$

System poles are zero points resulting from the subtraction between two transmissibility functions measured at the same output points i and j but with two different input excitation locations k and l.

$$\Delta T_{ij}^{kl}(s) = T_{ij}^k(s) - T_{ij}^l(s) \tag{3}$$

And consequently the poles of its inverse:

$$\Delta^{-1} T_{ij}^{kl}(s) = \frac{1}{T_{ij}^k(s) - T_{ij}^l(s)} \tag{4}$$

The PolyMAX method [23] is then investigated in order to calculate the system poles from $\Delta^{-1}T_{ij}^{kl}(s)$. Generally, additional poles can be present in the $\Delta^{-1}T_{ij}^{kl}(s)$ functions. Structural poles λ_r can easily be determined by performing a singular value decomposition of the transmissibility matrix $\mathbf{T}$ [19]. If we consider, for example, four different loading conditions k,l,m and n, the transmissibility matrix is the following:

$$\mathbf{T} = \begin{bmatrix} T_{1r}^k(s) & T_{1r}^l(s) & T_{1r}^m(s) & T_{1r}^n(s) \\ T_{2r}^k(s) & T_{2r}^l(s) & T_{2r}^m(s) & T_{2r}^n(s) \\ T_{3r}^k(s) & T_{3r}^l(s) & T_{3r}^m(s) & T_{3r}^n(s) \\ T_{4r}^k(s) & T_{4r}^l(s) & T_{4r}^m(s) & T_{4r}^n(s) \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{5}$$

In fact, in system poles λ_r the rank of matrix $\mathbf{T}$ is one; consequently, $\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4 \geqslant 0$ and $1/\sigma_2$ tends to ∞.

2.2. Stochastic subspace identification method (SSI)

The dynamic behavior of a discrete mechanical system consisting of n masses connected through springs and dampers is described by the following matrix differential equation:

$$\mathbf{M}\ddot{q}(t) + \boldsymbol{C_2}\dot{q}(t) + \mathbf{K}q(t) = f(t) \tag{6}$$

$\mathbf{M}$,$\boldsymbol{C_2}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$ are the mass, damping and stiffness matrices. $q(t) \in \mathbb{R}^n$ is the displacement vector at continuous time t. Vector $f(t) \in \mathbb{R}^n$ is the excitation force.

SSI is a method that converts a 2nd order problem into two 1st order problems. Eq. (6) can be converted into the following state equation:

$$\begin{aligned} \dot{x}(t) &= \boldsymbol{A_c}x(t) + \boldsymbol{B_c}f(t) \\ y(t) &= \mathbf{C}x(t) + \mathbf{D}f(t) \end{aligned} \tag{7}$$

The state matrix $\boldsymbol{A_c}$ in continuous time, the load matrix $\boldsymbol{B_c}$ and the output matrix $\mathbf{C}$ are given by:

$$\boldsymbol{A_c} = \begin{pmatrix} 0 & 11 \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\boldsymbol{C_2} \end{pmatrix}, \quad \boldsymbol{B_c} = \begin{pmatrix} 0 \\ \mathbf{M}^{-1} \end{pmatrix}, \quad \mathbf{C} = (11 \quad 0), \quad x = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}$$

$\mathbf{D}$ is the feedback matrix (zero in the case of mechanical systems). The first equation in (7) is called the state equation and models the dynamic behavior of the system. The second equation is called the observation or output equation. Eq. (7) can be converted to following discrete-time stochastic state-space model [8]:

$$\begin{aligned} x_{k+1} &= \mathbf{A}x_k + w_k \\ y_k &= \mathbf{C}x_k + v_k \end{aligned} \tag{8}$$

where $y_k = y(k\Delta t)$ is the sampled output vector, $x_k = x(k\Delta t)$ is the discrete state vector, w_k is the process noise due to the unknown excitation of the structure, v_k is the measurement noise and k is the time instant, $\mathbf{A} = exp(\boldsymbol{A_c}\Delta t)$ is the discrete state matrix. In order to obtain the modal parameters, an eigenvalue decomposition (EVD) of the matrix $\mathbf{A}$ is performed:

$$\mathbf{A} = \boldsymbol{\Psi}\boldsymbol{\Lambda_d}\boldsymbol{\Psi}^{-1} \tag{9}$$

$\boldsymbol{\Psi} \in \mathbb{C}^{n\times n}$ is the eigenvector matrix and $\boldsymbol{\Lambda_d} = diag(\lambda_i) \in \mathbb{C}^{n\times n}$ is the diagonal matrix containing the discrete time eigenvalues μ_i. The continuous time state Eq. (7) is equivalent to the second order matrix equation of motion (6). Consequently, they have the same eigenvalues