Plane wave formulas for spherical, complex and symplectic harmonics

H. De Bie, F. Sommen, M. Wutzig

PII: S0021-9045(17)30076-X
DOI: http://dx.doi.org/10.1016/j.jat.2017.07.001
Reference: YJATH 5157

To appear in: *Journal of Approximation Theory*

Received date : 19 December 2016
Revised date : 31 May 2017
Accepted date : 1 July 2017

Plane wave formulas for spherical, complex and symplectic harmonics

H. De Bie* F. Sommen† M. Wutzig‡

Department of Mathematical Analysis
Faculty of Engineering and Architecture – Ghent University
Galglaan 2, 9000 Gent, Belgium

Abstract

This paper is concerned with spherical harmonics, and two refinements thereof: complex harmonics and symplectic harmonics.

The reproducing kernels of the spherical and complex harmonics are explicitly given in terms of Gegenbauer or Jacobi polynomials. In the first part of the paper we determine the reproducing kernel for the space of symplectic harmonics, which is again expressible as a Jacobi polynomial of a suitable argument.

In the second part we find plane wave formulas for the reproducing kernels of the three types of harmonics, expressing them as suitable integrals over Stiefel manifolds. This is achieved using Pizzetti formulas that express the integrals in terms of differential operators.

Keywords: reproducing kernels, spherical harmonics, symplectic harmonics, Pizzetti formulas, plane waves, Stiefel manifolds

Mathematics Subject Classification: 32A50, 42B35

1 Introduction

In the study of harmonic analysis on the sphere (see e.g. [6]), spherical harmonics play a crucial role. Spherical harmonics of degree k are (the restrictions to the unit sphere of) harmonic polynomials of homogeneity k. In many applications, one is not so much interested in the harmonics themselves, but rather in their reproducing kernels. For spherical harmonics, the reproducing kernel is given in terms of Gegenbauer polynomials (see [16] and subsequent Theorem 2).

Several refinements of spherical harmonics have been introduced over the last decades. Koornwinder (see [9]) defined complex harmonics of degree (p, q) as spherical harmonics which are homogeneous of order p in the complexified variables and of order q in the complex conjugated variables. He determined the reproducing kernel of the complex harmonics in terms of a Jacobi polynomials (see also subsequent Theorem 4), which was the first step in establishing his celebrated addition formula for the Jacobi polynomials.

Recently, a further refinement of complex harmonics was introduced in [3]. They are called symplectic harmonics, as the space they span is invariant under the symplectic group. In addition to being (p, q) complex harmonics, symplectic harmonics have to be in the kernel of a certain twisted Euler operator (see the subsequent Definition 1).

Let us now describe the main results of our paper. First we determine the reproducing kernel of the

*E-mail: Hendrik.DeBie@UGent.be
†E-mail: Frank.Sommen@UGent.be
‡E-mail: Michael.Wutzig@UGent.be