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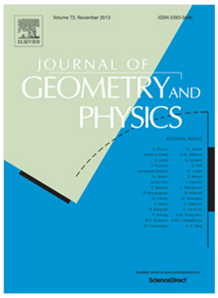

Rigidity of complete noncompact Riemannian manifolds with harmonic curvature

Bingqing Ma, Guangyue Huang

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RIGIDITY OF COMPLETE NONCOMPACT RIEMANNIAN MANIFOLDS WITH HARMONIC CURVATURE

BINGQING MA AND GUANGYUE HUANG

ABSTRACT. For complete noncompact Riemannian manifolds (M^n, g) with harmonic curvature, we prove that g is Einstein under an inequality involving $L^{\frac{n}{2}}$-norm of the Weyl curvature, the traceless Ricci curvature and the Sobolev constant. Furthermore, we achieve that M^n is a constant curvature space under such inequality and finite L^2-norm of the Weyl curvature.

MSC (2010). Primary 53C24, Secondary 53C21.
Keywords: Sobolev constant, harmonic curvature, rigidity.

1. Introduction

Recently, Catino, in [3], investigates compact gradient shrinking Ricci solitons (M^n, g) satisfying a $L^{\frac{n}{2}}$-pinching condition, he proves that M^n is isometric to a quotient of the round $\mathbb{S}^n$. The proof relies mainly on sharp algebraic curvature estimates and an improved rigidity results for integral pinched Einstein metrics. Inspired by Catino's idea, the authors in [12] studied rigidity of compact manifolds with Bach-flat tensor. In this paper, we continue to study complete noncompact manifolds with harmonic curvature and achieve some similar rigidity results.

We recall that a Riemannian manifold (M^n, g) is a manifold with harmonic curvature if the divergence of the Riemannian curvature vanishes (that is, $R_{ijkl,l} = 0$). By virtue of the second Bianchi identity, we have

$$R_{ki,j} - R_{kj,i} = R_{ijkl,l} \tag{1.1}$$

which shows that the Ricci curvature of a manifold with harmonic curvature is a Codazzi tensor (a Codazzi tensor is a (2,0)-symmetric tensor field T satisfying $T_{ij,k} = T_{ik,j}$ for any i, j, k). Contracting the index i, k in (1.1) for manifolds with harmonic curvature yields

$$R_{ij,i} = R_{,j}, \tag{1.2}$$

which combining with the second Bianchi identity

$$R_{ij,i} = \frac{1}{2} R_{,j}, \tag{1.3}$$

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