Optics Communications 414 (2018) 146–153

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Kerr-like behaviour of second harmonic generation in the far-off resonant regime

Vlasta Peřinová [a], Antonín Lukš [a], Jaromír Křepelka [a,*], Wiesław Leoński [b], Jan Peřina Jr. [c]

[a] RCPTM, Joint Laboratory of Optics of Palacký University and Institute of Physics of Czech Academy of Sciences, Faculty of Science, Palacký University, 17. listopadu 12, 771 46 Olomouc, Czech Republic

[b] Quantum Optics and Engineering Division, Institute of Physics, University of Zielona Góra, Prof. Z. Szafrana 4a, 65-516 Zielona Góra, Poland

[c] Institute of Physics of Czech Academy of Sciences, Joint Laboratory of Optics of Palacký University and Institute of Physics, 17. listopadu 50a, 771 46 Olomouc, Czech Republic

ARTICLE INFO

Keywords:
Second-harmonic generation
Kerr-like behaviour
Quantum nonlinear optics
Floquet-type decomposition
Superposition states

ABSTRACT

We separate the Kerr-like behaviour of the second-harmonic generation in the far-off resonant regime from the oscillations caused by the time-dependence of the interaction energy. To this purpose, we consider the approximation obtained from the exact dynamics by the method of small rotations. The Floquet-type decomposition of the approximate dynamics comprises the Kerr-like dynamics and oscillations of the same order of magnitude as those assumed for the exact dynamics of the second-harmonic generation. We have found that a superposition of two states of concentrated quantum phase arises in the fundamental mode in the second-harmonic generation in the far-off resonant limit at a later time than a superposition of two coherent states in the corresponding Kerr medium and the difference is larger for higher initial coherent amplitudes.

The quantum phase fluctuation is higher for the same initial coherent amplitudes in the fundamental mode in the second-harmonic generation in the far-off resonant limit than in the corresponding Kerr medium and the difference is larger for higher initial coherent amplitudes.

1. Introduction

The second- and third-order nonlinear media were studied not only in nonlinear, but also in quantum optics. The refractive index of the third-order nonlinear medium can depend on the intensity of light. This dependence is known as the optical Kerr effect and such a medium is called a Kerr medium. In quantum optics, the squeezing of light was investigated first [1–3]. Later, the attention was paid to the superposition states of the light modes [4,5]. The third-order nonlinear isotropic medium is called also a Kerr medium. The light mode in the Kerr medium is modelled as an anharmonic oscillator. The initial coherent state of this oscillator evolves into a generalized coherent state. The conditions on which the generalized coherent state is a superposition of coherent states were studied theoretically [6].

The superposition states are of fundamental interest [7,8] and, to the different methods for generating such states, including superpositions of squeezed states, the Kerr nonlinearities belong [9].

From the viewpoint of nonlinear and applied optics it is important that the cascaded second-order effects (second-harmonic generations) behave as a third-order nonlinear process [10–12]. In quantum optics a Kerr-like behaviour has been derived from the solution of the description of second-harmonic generation in the case of mismatch up to the second order in the propagation distance [13,14]. Equivalently, the derivation can proceed up to the second order in the coupling constant [15,16].

A driven mode in the Kerr medium or in the medium with effectively Kerr behaviour has interesting properties. The one-photon blockade has been invented [17] and the subsequent investigation of the finite-dimensional states engineering has been successful [18]. Also the notions of two- and three-photon blockades are useful [19].

The outline of the paper is as follows. In Section 2, we present a formal derivation of the Kerr-like behaviour of the second-harmonic generation in the far-off resonant regime. In Section 3, we mention the invariant-subspace method. In Section 4, using numerical results, we illustrate the Kerr-like behaviour of various characteristics of the second-harmonic generation and deviations from this behaviour. Especially, we deal with the fidelity between the fundamental mode and the light in the Kerr medium. In Section 5, we conclude.

* Corresponding author.
E-mail address: jaromir.krepelka@upol.cz (J. Křepelka).

https://doi.org/10.1016/j.optcom.2018.01.017
Received 10 October 2017; Received in revised form 8 January 2018; Accepted 9 January 2018
0030-4018/

2. Kerr-like behaviour of second harmonic generation

The second harmonic generation is usually described on the condition of perfect resonance [20], but without this assumption [16], we must use the following model Hamiltonian (in units $\hbar = 1$)

$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm int} \tag{1}$$

$$= \sum_{j=1}^{2} \omega_j \hat{a}_j^\dagger \hat{a}_j + g\hat{a}_1^2\hat{a}_2^\dagger + g^*\hat{a}_1^{\dagger 2}\hat{a}_2, \tag{2}$$

where $\hat{a}_1$ ($\hat{a}_1^\dagger$) and $\hat{a}_2$ ($\hat{a}_2^\dagger$) are annihilation (creation) operators of the fundamental mode of frequency ω_1 and of the second harmonic mode of frequency ω_2, respectively. It holds that $\omega_2 = 2\omega_1$ on resonance. The constant g describes coupling between both modes.

In the Schrödinger picture, the modal operators do not change, $\hat{a}_j = \hat{a}_j(0)$, $j = 1, 2$, but the state vector $|\psi(t)\rangle$ evolves. On the replacement $\exp(i\hat{H}_0 t)|\psi(t)\rangle \to |\psi(t)\rangle$, we obtain the interaction picture without introducing new notation. The evolution of the state vector $|\psi(t)\rangle$ in the interaction representation can be described by the equation

$$\frac{d}{dt}|\psi(t)\rangle = -i\hat{\mathcal{H}}_{\rm int}(t)|\psi(t)\rangle, \tag{3}$$

where

$$\hat{\mathcal{H}}_{\rm int}(t) = e^{i\hat{H}_0 t}\hat{H}_{\rm int}e^{-i\hat{H}_0 t} \tag{4}$$

$$= g\hat{a}_1^2\hat{a}_2^\dagger \exp(i\Delta\omega t) + g^*\hat{a}_1^{\dagger 2}\hat{a}_2 \exp(-i\Delta\omega t), \tag{5}$$

with $\Delta\omega = \omega_2 - 2\omega_1$ being detuning. Eq. (3) has the solution

$$|\psi(t)\rangle = \hat{U}_{\rm int}(t)|\psi(0)\rangle, \tag{6}$$

where $\hat{U}_{\rm int}(t)$ is the solution of the initial-value problem

$$\frac{d}{dt}\hat{U}_{\rm int}(t) = -i\hat{\mathcal{H}}_{\rm int}(t)\hat{U}_{\rm int}(t),\ \hat{U}_{\rm int}(0) = \hat{1}. \tag{7}$$

To solve the problem (7), we make a suitable substitution and obtain

$$\hat{U}_{\rm int}(t) = \hat{U}_{\rm pri}(t)\hat{U}_{\rm nonpri}(t), \tag{8}$$

where

$$\hat{U}_{\rm pri}(t) = e^{i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2},\ \hat{U}_{\rm nonpri}(t) = e^{-it\hat{H}_{\rm red}}, \tag{9}$$

with

$$\hat{H}_{\rm red} = \Delta\omega\hat{a}_2^\dagger\hat{a}_2 + \hat{H}_{\rm int}, \tag{10}$$

where red means reduced. Eq. (3) has $\frac{2\pi}{\Delta\omega}$-periodic coefficients in the number-state basis and, therefore, the Floquet theory [21] can be applied. In relation (8), the subscript pri means a $\frac{2\pi}{\Delta\omega}$-periodic factor and the subscript nonpri means an aperiodic factor. This decomposition does not seem to have another physical meaning besides the exact solution of the problem (7).

Using the formalism in [16], we derive an approximate evolution operator in the interaction picture, $\hat{\hat{U}}_{\rm int}(t) \approx \hat{U}_{\rm int}(t)$,

$$\hat{\hat{U}}_{\rm int}(t) = e^{i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2}\hat{U}^\dagger \times e^{-i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2 - it\frac{|g|^2}{\Delta\omega}[\hat{X}_+,\hat{X}_-]}\hat{U}, \tag{11}$$

where

$$\hat{X}_+ = \hat{a}_1^2\hat{a}_2^\dagger,\ \hat{X}_- = \hat{a}_1^{\dagger 2}\hat{a}_2,$$
$$\hat{U} = \exp\left[\frac{1}{\Delta\omega}(g\hat{X}_+ - g^*\hat{X}_-)\right]. \tag{12}$$

Let us recall that

$$[\hat{X}_+, \hat{X}_-] = 4\hat{n}_1\hat{n}_2 + 2\hat{n}_2 - \hat{n}_1^2 + \hat{n}_1, \tag{13}$$

where $\hat{n}_j = \hat{a}_j^\dagger\hat{a}_j$, $j = 1, 2$. We introduce also an approximate evolution operator $\hat{\hat{\hat{U}}}_{\rm int} \approx \hat{\hat{U}}_{\rm int}(t) \approx \hat{U}_{\rm int}(t)$,

$$\hat{\hat{\hat{U}}}_{\rm int}(t) = e^{i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2}\hat{U}^\dagger \times e^{-i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2}\hat{U}e^{-it\frac{|g|^2}{\Delta\omega}[\hat{X}_+,\hat{X}_-]}, \tag{14}$$

where we have calculated to the second order in $\frac{|g|}{\Delta\omega}$. We apply the Floquet theory to relation (14) as follows. We find easily that

$$\hat{\hat{\hat{U}}}_{\rm int}\left(\frac{2\pi}{\Delta\omega}\right) = e^{-i2\pi\frac{|g|^2}{(\Delta\omega)^2}[\hat{X}_+,\hat{X}_-]}. \tag{15}$$

From this, we introduce

$$\hat{\hat{\hat{U}}}_{\rm nonpri}(t) = \left[\hat{\hat{\hat{U}}}_{\rm int}\left(\frac{2\pi}{\Delta\omega}\right)\right]^{\frac{\Delta\omega t}{2\pi}} = e^{-it\frac{|g|^2}{\Delta\omega}[\hat{X}_+,\hat{X}_-]} \tag{16}$$

and

$$\hat{\hat{\hat{U}}}_{\rm pri}(t) = \hat{\hat{\hat{U}}}_{\rm int}(t)\hat{\hat{\hat{U}}}^\dagger_{\rm nonpri}(t) = e^{i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2}\hat{U}^\dagger e^{-i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2}\hat{U}. \tag{17}$$

The function (17) is really $\frac{2\pi}{\Delta\omega}$-periodic. According to (17), it holds that

$$\hat{\hat{\hat{U}}}_{\rm int}(t) = \hat{\hat{\hat{U}}}_{\rm pri}(t)\hat{\hat{\hat{U}}}_{\rm nonpri}(t). \tag{18}$$

The decomposition (18) of the approximate evolution operator (14) has the following physical meaning. Let us assume that, initially, the fundamental frequency mode is in any single-mode state and the second harmonic mode is in a vacuum state. The operator $\hat{\hat{\hat{U}}}_{\rm nonpri}(t)$ means the effectively Kerr dynamics. The operator $\hat{\hat{\hat{U}}}_{\rm pri}(t)$ means a deviation from this dynamics. Since the operator $\hat{\hat{\hat{U}}}_{\rm pri}(t)$ itself has been derived in an approximate calculation, it is not the deviation of the exact evolution from the effectively Kerr dynamics. Using the numerical examples below we will show that $\hat{\hat{\hat{U}}}_{\rm pri}(t)$ leads only to order-of-magnitude coincidences with the exact dynamics.

As was said above, Eq. (11) is derived similarly as the effective Hamiltonian is derived from the interaction one in [16]. Let us note two minor differences:

(1) In [16], the effective Hamiltonian is expressed as a transform of the interaction Hamiltonian, but for the purposes of this paper on the contrary the reduced Hamiltonian should be expressed as a transform of the effective Hamiltonian.

(2) In [16], the interaction Hamiltonian is rather a version of the reduced Hamiltonian and as such it should be denoted as

$$\hat{H}_{\rm redKSS} = \Delta\omega\hat{X}_3 + \hat{H}_{\rm int}, \tag{19}$$

where KSS means Klimov and Sánchez-Soto and $\hat{X}_3 = \frac{1}{3}(\hat{a}_2^\dagger\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_1)$. The similarity of the derivation is given by the relation

$$\hat{a}_2^\dagger\hat{a}_2 = \hat{X}_3 + \frac{1}{3}\hat{N}, \tag{20}$$

where

$$\hat{N} = \hat{a}_1^\dagger\hat{a}_1 + 2\hat{a}_2^\dagger\hat{a}_2 \tag{21}$$

is a constant of motion.

The approaches in [16] and in this paper are able to express the exact evolution of the state vector in the Schrödinger picture. The first approach is simpler,

$$\exp(-it\hat{H})|\psi(0)\rangle = \exp\left(-it\frac{\omega_1+\omega_2}{3}\hat{N}\right)\exp(-it\hat{H}_{\rm redKSS})|\psi(0)\rangle. \tag{22}$$

The product of the last two factors on the right-hand side describes the evolution in the picture of the first approach. The second approach is a little more complicated,

$$\exp(-it\hat{H})|\psi(0)\rangle = \exp(-it\hat{H}_0) \times \exp(i\Delta\omega t\hat{a}_2^\dagger\hat{a}_2)\exp(-it\hat{H}_{\rm red})|\psi(0)\rangle. \tag{23}$$

The product of the last three factors on the right-hand side describes the evolution in the picture of the second approach. On equating, it follows that

$$\exp(-it\hat{H}_{\rm redKSS})|\psi(0)\rangle = \exp(-i\Delta\omega t\hat{X}_3)$$