



# Harmonic Balance method for nonlinear and viscous free surface flows

Inno Gatin<sup>a,\*</sup>, Gregor Cvijetić<sup>a</sup>, Vuko Vukčević<sup>a</sup>, Hrvoje Jasak<sup>a,b</sup>, Šime Malenica<sup>c</sup>

<sup>a</sup> University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Ivana Lučića 5, Zagreb, Croatia

<sup>b</sup> Wikki Ltd, 459 Southbank House, SE1 7SJ, London, United Kingdom

<sup>c</sup> Bureau Veritas Marine & Offshore, Departement Recherche, Le Triangle de l'Arche, 8 Cours du Triangle - CS 50101, 92937 Paris La Defense Cedex, France

## ARTICLE INFO

### Keywords:

Periodic  
Free surface flows  
CFD  
Harmonic Balance  
Naval hydrodynamics  
Foam–extend

## ABSTRACT

The Harmonic Balance method applied to fully nonlinear, viscous, temporally periodic free surface flows is presented in this paper. The Harmonic Balance approach is used to project a transient periodic two phase flow into multiple coupled steady–state problems discretising the periodic time interval. The numerical framework is based on Finite Volume method for Computational Fluid Dynamics in the open–source software foamextend. The disability of the Harmonic Balance method to simulate flows with zero or close to zero mean velocity is overcome by coupling the Harmonic Balance steady–state equations implicitly, presenting a novelty in the field. Von Neumann stability analysis is performed for the governing Harmonic Balance equations to mathematically prove that zero mean velocity can be simulated using the proposed implicit coupling. The method is validated and verified on a periodic free surface flow over a ramp and regular surface wave propagation with current, both including grid convergence studies and spectral resolution sensitivity studies. All results are compared to fully transient computations. A detailed study is performed for wave propagation without current, confirming that convergence of free surface elevation can be achieved without mean velocity. The approach proved accurate and applicable for twophase flows, opening a new research field.

## 1. Introduction

This work presents the development, implementation and validation of the time–spectral method for simulating fully nonlinear, viscous, temporally periodic, large scale two phase flows, based on an existing two phase numerical model developed within a Finite Volume (FV) Computational Fluid Dynamics (CFD) software foam–extend (Jasak, 2009). The method is specifically intended for general two–phase flows where the boundary conditions have a temporally periodic nature with low or zero mean velocity, resulting in a periodic flow field in the domain of interest. Such flows are often encountered in problems regarding surface waves which are important in the field of naval, offshore and ocean hydrodynamics.

The frequency domain methods are very common in the naval hydrodynamics, but mainly within the potential flow approach. The simplest example is the evaluation of the linear transfer functions (so called Response Amplitude Operator, RAO) which is the basic ingredient of the linear spectral analysis which allows for very quick assessment of the floating body response for any particular sea conditions. The main drawback of the frequency domain approach, within the potential flow

assumptions, is associated with the difficulties related to the inclusion of the different nonlinear effects. Indeed, within the classical approach the resulting boundary value problems for higher order velocity potentials quickly become very complex leading to many numerical difficulties (e.g. see Ferrant et al. (1999), Malenica and Molin (1995)). Consequently, the common practice within the potential flow approach is to resolve up to the second order in frequency domain and apply the fully nonlinear time domain approach for highly nonlinear problems. However, it is well known that the potential flow approach has very limited use for fully nonlinear wave body interactions because of the difficulties associated with the proper treatment of the complex free surface boundary conditions. That is why there is still no efficient fully nonlinear wave body interaction solver based on potential flow theory which has been proposed in the literature. Only some relatively simple nonlinear wave body interaction problems, such as one discussed in Ferrant et al. (1999), were efficiently solved within the potential flow approach. Viscous CFD methods are an obvious alternative, since they offer reliable results, as shown in numerous validation publications (Vukčević et al., 2015, 2016a; Vukčević and Jasak, 2015a; Gatin et al., 2015; Larsson et al., 2013). Consequently, in the recent years a number of publications

\* Corresponding author.

E-mail addresses: [inno.gatin@fsb.hr](mailto:inno.gatin@fsb.hr) (I. Gatin), [gregor.cvijetic@fsb.hr](mailto:gregor.cvijetic@fsb.hr) (G. Cvijetić), [vuko.vukcevic@fsb.hr](mailto:vuko.vukcevic@fsb.hr) (V. Vukčević), [h.jasak@wikki.co.uk](mailto:h.jasak@wikki.co.uk), [hrvoje.jasak@fsb.hr](mailto:hrvoje.jasak@fsb.hr) (H. Jasak), [sime.malenica@bureauveritas.com](mailto:sime.malenica@bureauveritas.com) (Š. Malenica).

<https://doi.org/10.1016/j.oceaneng.2018.03.050>

Received 8 September 2017; Received in revised form 4 January 2018; Accepted 18 March 2018

dealing with the application of viscous CFD on the problems of wave induced motions and loads in naval hydrodynamics have emerged (Silva et al., 2017; Hashimoto et al., 2016; Wu et al., 2011; Thilleul et al., 2011; Kim et al., 2012; Chen et al., 2014; Windén et al., 2012; Mousaviraad et al., 2015; Carrica et al., 2011; Kim, 2011; Qiu et al., 2014; Carrica et al., 2008; Seng et al., 2014; el Moctar et al., 2017).

Time domain simulations prevail when it comes to viscous naval hydrodynamic CFD calculations. However, in the field of turbomachinery, spectral CFD methods present an active area of research. Nonlinear harmonic method applied to Navier–Stokes equations was first presented by He and Ning (1998) for simulating unsteady viscous flow around turbomachinery blades. The nonlinear harmonic method, also called Harmonic Balance (HB), takes advantage of the temporally periodic nature of the flow with a known dominant frequency of oscillation by transforming a transient problem into a number of coupled steady state problems. The main drawback of simulating periodic flows with viscous CFD using transient time–marching techniques is the requirement for large number of simulated periods before reaching fully developed periodical flow (Simonsen et al., 2013a; Vukčević and Jasak, 2015a; Larsson et al., 2015a, 2015b). HB alleviates this problem, while additionally accelerating the calculation by considering steady state problems which are generally faster to perform. The advantages of HB were soon recognized in the CFD community, resulting in a large number of publications mostly related to turbomachinery applications. Maple et al. (2004) developed an adaptive HB method where the number of resolved harmonics is varied in the domain depending on the required spectral resolution, applied to a supersonic/subsonic diverging nozzle. McMullen and Jameson (2005) investigated the acceleration techniques for solving the coupled set of steady state problems, where they reported 8–19 times acceleration in terms of computational time comparing with the transient code. Ekici et al. (2008) used Euler equations based HB method to simulate helicopter rotor blade flow, while Guédeney et al. (Guédeney et al., 2013) extended the method for turbomachinery flows with multiple frequencies in order to capture rotor–stator interaction effects.

Apart from being easier and faster to calculate, steady state equations provide the ability of automatic optimisation. In case of HB method, objectives for optimisation can be related to periodic quantities. Huang (Huang and Ekici, 2014) used HB method in conjunction with discrete adjoint optimisation method to optimise turbomachinery blades. In the field of naval hydrodynamics, minimisation of added wave resistance or ship motion could be achieved.

To the authors' knowledge, no attempt has been made to develop a HB method for nonlinear, two–phase, viscous and turbulent free surface flows with low or zero mean flow velocity. In this work HB method is employed for the calculation of periodic free surface flows, where implicit coupling of steady–state equations is performed by solving the equations in a block matrix form. Implicit coupling enables running simulations at large reduced frequency numbers (Hall et al., 2013), which is the stability criteria stemming from the von Neumann stability analysis of the HB equations. In practice, this enables low and zero mean flow velocities, and coarser spatial grids. The implicit coupling presents a novelty with respect to existing publications regarding the HB method. The development is based on the previously performed implementation by Cvijetić et al. (2016), where single–phase, explicit HB method is described for turbomachinery applications.

The targeted practical applications of the presented HB method are associated with the evaluation of the transfer functions of ship's response in waves at different orders of approximation (linear, quadratic, higher order etc.) and for assessing added resistance in waves. These transfer functions are then used in combination with the body operating conditions to derive the long term design values (e.g. see Journée and Massie (Journée and Massie, 2001)). Even though the real sea state is irregular and composed of very large number of individual waves with different amplitude and frequencies, design methods exist which are based on the use of the so called equivalent regular design waves (Soares et al., 2015).

Hence, having the efficient numerical tool for the evaluation of the wave body interactions in regular waves can be very useful in practice.

This paper is organised as follows: first the mathematical model is presented, comprising the governing equations in the SWENSE (Spectral Wave Explicit Navier–Stokes Equations) decomposed form as described by Vukčević et al. (2016b) and the HB treatment of the time derivative term. Next, an overview of the numerical method is given including the implicit coupling of the HB source term, followed by brief notes on implementation of the Ghost Fluid Method (GFM) in foam–extend (Vukčević et al., 2017). In order to assess the new model, two validation test cases are presented: a two–phase periodic flow over a ramp and regular wave propagation with uniform current. Additionally, the implicit method is compared to explicitly coupled HB method for the wave propagation case with current. Next, a stability study is performed on a wave propagation case without current. Finally, concluding remarks are given.

## 2. Mathematical model

In this section, a brief overview of the mathematical model describing two–phase, nonlinear and viscous flow is given, presenting the governing equations in the SWENSE (Vukčević et al., 2016b; Ducroz et al., 2014; Ferrant et al., 2002) decomposed form, and briefly outlining the formulation of dynamic pressure and density jump at the interface using the GFM (Huang et al., 2007; Vukčević et al., 2017). Furthermore, a description of governing equations in the time–spectral HB form is given.

### 2.1. Governing equations

Two–phase, incompressible, viscous flow is governed by the momentum conservation equation, mass conservation equation and an interface capturing equation in the appropriate form. The equations below are given in the SWENSE decomposed form (Vukčević et al., 2016b), where an arbitrary field  $q$  is written as  $q = q_I + q_P$ . Index  $I$  denotes the incident component, typically corresponding to a solution obtained by potential flow, while index  $P$  denotes the perturbation component being solved for via equation set. The incident terms are treated explicitly, as they are assumed to be known at each time instant. Formally, this decomposition is arbitrary, but it is assumed that the incident field provides a good estimate of the final nonlinear solution, i.e. that the component  $q_I$  is smaller than  $q_P$ . Reader is directed to (Vukčević et al., 2017) for detailed derivation of the following equation set.

For incompressible flow the conservation of mass reads:

$$\nabla \cdot \mathbf{u}_P = -\nabla \cdot \mathbf{u}_I, \quad (1)$$

where  $\mathbf{u}$  denotes the velocity field. Although the potential flow solution is formally divergence free,  $\nabla \cdot \mathbf{u}_I = 0$ , mapping this field to a discretised volume mesh generally introduces continuity errors (Vukčević et al., 2016b) which are resolved in Eqn. (1).

The SWENSE decomposed momentum equation reads:

$$\frac{\partial(\vec{u}_P)}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}_P) - \nabla \cdot (\nu_{eff} \nabla \mathbf{u}_P) = -\frac{\partial(u_I)}{\partial t} - \nabla \cdot (\mathbf{u}\mathbf{u}_I) + \nabla \cdot (\nu_{eff} \nabla \mathbf{u}_I) - \frac{1}{\rho} \nabla p_d + \nabla \mathbf{u} \cdot \nabla \nu_{eff}, \quad (2)$$

where  $\nu_{eff}$  stands for effective kinematic viscosity including the turbulent viscosity,  $\rho$  is the density which has a discontinuity at the interface, while  $p_d$  stands for dynamic pressure:  $p_d = p - \rho \mathbf{g} \cdot \mathbf{x}$ , where  $p$  denotes total pressure,  $\mathbf{g}$  represents constant gravitational acceleration, and  $\mathbf{x}$  is the radii vector. The velocity in the last term on the right hand side is not decomposed since it is treated explicitly. More details regarding decomposition of the pressure term are given in (Vukčević et al., 2016b).

In this study, Level Set (LS) method derived from the Phase Field (PF) equation (Sun and Beckermann, 2007, 2008) is used for interface

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات