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High efficiency second and third harmonic generation from magnetic metamaterials by using a grating

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ABSTRACT

Metamaterials can be used to generate harmonic signals in small thicknesses, but they suffer from low efficiency. Here, we introduce a new method for amplifying second and third harmonic generation from magnetic metamaterials. We show numerically that by using a grating structure under the metamaterial, the grating and the metamaterial form a resonator which leads to a higher absorption in the metamaterial. By this method we could increase the absorption of the structure in the magnetic resonance up to 25% of the initial value. This leads to the generation of second and third harmonic signals with a higher efficiency from this metamaterial-based nonlinear media. We confirmed this idea in the nanostrip metamaterials and saw the amplitude of the second harmonic generation was doubled and the amplitude of the third harmonic generation increased by a factor of 4 in comparison to the same structure without grating.

1. Introduction

Metamaterials have achieved many breakthroughs in today's optics by their special properties, such as negative refractive index [1,2], invisibility cloaking [3,4], super-resolution imaging [5,6], metasurfaces [7,8]. Many nonlinear effects have also been shown and proposed based on metamaterials, like using nonlinear photonic elements such as diodes for generating nonlinear effects [9,10], second and third harmonic generation [11,12], solitons [13], and tunable nonlinear metamaterials by using liquid crystals [14]. Also some theoretical modelling of nonlinear plasmonics have done recently [15–18]. In this paper, we introduce a highly efficient method for generating second and third harmonic signals from metamaterials. This is achieved by including a grating structure below the metamaterial. The grating and the metamaterial form a Fabry-Perot resonator which leads to an enhancement of resonating behavior of the metamaterial. At first, we show theoretically and numerically the effect of grating on the absorption of the metamaterial and how it enhances the absorption of the structure. The theoretical method is the transfer matrix method (TMM) which is a semi-analytical method for evaluating the transmittance and reflectance of multilayer structures. We show numerically that the enhanced absorption is equivalent to the enhanced local induced currents. The induced currents in the metallic parts are shown to be the source of nonlinear effects in the magnetic metamaterials [19,20]. Therefore, increased polarization currents mean more powerful nonlinear effects. The amount of amplification of nonlinear effects is investigated numerically and is compared to the same structure without using the grating.

2. Theoretical background

Magnetic metamaterials show their nonlinear behavior in their magnetic resonance. In order to find the resonance regions, we should obtain the metamaterial absorption spectra (see Fig. 1(a)). To do this we illuminated the structure with a TM polarized wave. The TM polarization insures the magnetic response of the structure. In the TM polarization the magnetic field is normal to the cross section of the structure. As is shown in the inset of Fig. 1(b) the structure can be visualized as an LC circuit, in which the somehow round shape of the metamaterial acts as the self-inductance. When the magnetic field is normal to this self-inductance, a current loop is generated in the metallic parts to resist the passing flux due to the Lenz's law (see Fig. 1(b)). This leads to a magnetic dipole which reaches its maximum at magnetic resonance frequency (the resonance frequency of LC circuit). In Fig. 1(b) the black arrows show the polarization currents and the blue area is the magnetic dipole. The full details about the LC

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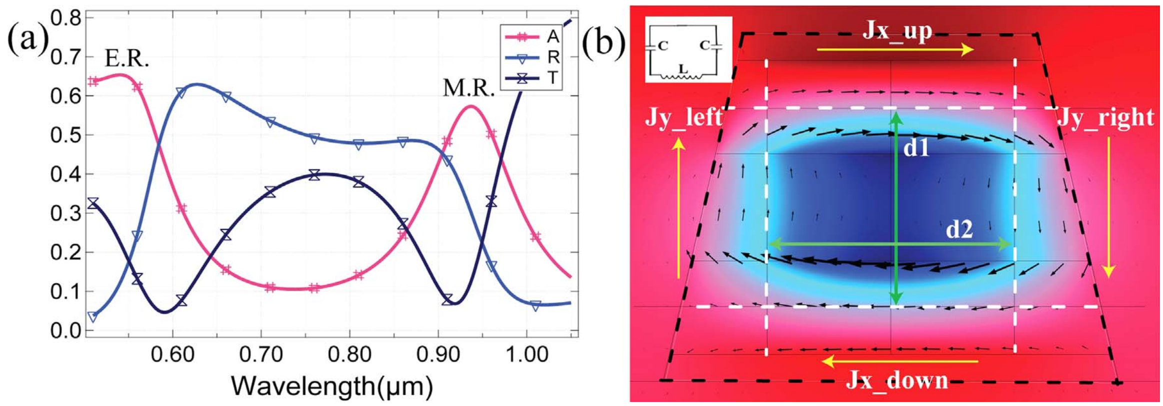

Fig. 1. (a) Transmission, reflection and absorption spectra of nanostrip without grating. The peaks in the absorption curve determine the electric resonance (E.R.) and the magnetic resonance (M.R.) frequencies. (b) Magnetic field distribution and electric field displacement (black arrows) in magnetic resonant frequency of nanostrip without grating. The external magnetic field induces a current loop which can be divided into two orthogonal groups. The current loop components are measured on the white dashed lines. The inset shows the equivalent circuit, and the metamaterial is denoted by the black dash box.

representation of magnetic metamaterials can be found in [1,2]. We focus on this phenomenon in here and try to increase these induced polarization currents which later leads to stronger nonlinear effects. We illustrate our idea of highly efficient nonlinear signal generation, by focusing on the magnetic resonance of nanostrip metamaterials [21].

The maxima in the absorption spectra give the resonance frequencies (see Fig. 1(a)). For easier calculation of scattering parameters (transmission and reflection) and also the nonlinear parameters, we used the normal illumination (zero angle of incidence). To find the resonance frequencies we need to do a broad wavelength sweep. The wavelength range that we used was 500 nm to 1050 nm. It turned out that the structure shows electric resonance at around 550 nm and magnetic resonance at around 920 nm (see Fig. 1(a)). Both parts of Fig. 1 are related to the metamaterial without the grating. As we will show, adding the grating will increase the absorption of the metamaterial and also the polarization currents induced in the metallic parts (the black arrows in Fig. 1(b)) and the overall curves and field distribution will not change too much. So, we showed the fundamental properties of the metamaterial in Fig. 1 and we discuss the effect of grating on absorption and currents in the remaining figures.

First, we investigate the effect of the grating on the metamaterial absorption. In order to see the effect of the grating on the metamaterial, we use the transfer matrix method (TMM). TMM is a semi-analytical method for investigating multilayer structures [22]. In this method, a scattering matrix is defined for each layer. A scattering matrix gives us information about scattering properties (absorption, transmittance and reflection) of that layer. For a multilayer structure, a scattering matrix is defined for each layer, and the structure's scattering matrix is obtained by combining these matrices. The definition of transfer matrix and the combination method is as follows. Transfer matrix for the ith layer of a multilayer structure is:

$$\begin{pmatrix} E_L^+ \\ E_R^- \end{pmatrix} = \begin{pmatrix} S_{11}^{(i)} & S_{12}^{(i)} \\ S_{21}^{(i)} & S_{22}^{(i)} \end{pmatrix} \begin{pmatrix} E_L^- \\ E_R^+ \end{pmatrix} \tag{1}$$

where E_L and E_R are the electric field in the left and right boundaries of the layer and +, - show the direction of propagation.

If we have a scattering matrix $S^{(A)}$ that is to be followed by a scattering matrix $S^{(B)}$(layer A followed by layer B), the combined scattering matrix can be calculated as follows:

$$\begin{pmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{pmatrix} = \begin{pmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{pmatrix} \otimes \begin{pmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{pmatrix} \tag{2}$$

Where the star product is defined as follows:

$$S_{11}^{(AB)} = S_{11}^{(A)} + S_{12}^{(A)}[I - S_{11}^{(B)}S_{22}^{(A)}]^{-1}S_{11}^{(B)}S_{21}^{(A)} \tag{3}$$

$$S_{12}^{(AB)} = S_{12}^{(A)}[I - S_{11}^{(B)}S_{22}^{(A)}]^{-1}S_{12}^{(B)} \tag{4}$$

$$S_{21}^{(AB)} = S_{21}^{(B)}[I - S_{22}^{(A)}S_{11}^{(B)}]^{-1}S_{21}^{(A)} \tag{5}$$

$$S_{22}^{(AB)} = S_{22}^{(B)} + S_{21}^{(B)}[I - S_{11}^{(B)}S_{22}^{(A)}]^{-1}S_{12}^{(B)}S_{22}^{(A)} \tag{6}$$

These scattering parameters should be calculated numerically. This means that the structure should be radiated twice from left and right, and so 4 scattering parameters are calculated for each structure. In our model, we defined the nanostrips as the first layer and the grating and the space between the grating and nanostrip as the second layer. It should be mentioned that an empty medium or a medium with a constant refractive index can be calculated with a propagation matrix separately as follows:

$$P_i = \begin{pmatrix} e^{ik_i x} & 0 \\ 0 & e^{-ik_i x} \end{pmatrix} \tag{7}$$

In other words, we can either consider two matrices for the separating layer and the grating itself or we can use just one matrix for both media. For example, if the grating matrix is denoted by G, we can define the transfer matrix of separating layer and grating denoted by $S^{(B)}$ as follows:

$$S^{(B)} = P \otimes G \tag{8}$$

After calculating the scattering matrix, the obtained absorption of the structure is defined as follows:

$$A = 1 - R - T = 1 - |S_{21}^{(AB)}|^2 - |S_{21}^{(AB)}|^2 \tag{9}$$

The result of this calculation is shown in Fig. 2(b).

The nonlinear behavior of metamaterials is originated form the nonlinear behavior of metals, and nonlinear behavior of the dielectric parts is much smaller than metals in nano thicknesses [11,23]. To see the effect of currents on generating nonlinear signals in metals, we investigate the linear and nonlinear behavior of the metals. The linear behavior of the metals can be obtained by using the Drude model, which is the equation of motion of free electrons in metals [24]:

$$m\frac{\partial^2 \vec{r}(t)}{\partial t^2} + m\Gamma\frac{\partial \vec{r}(t)}{\partial t} = -e\vec{E_0}e^{-i\omega t} \tag{10}$$

In the above formula, the right-hand side term is the driving force caused by the exciting wave, and the middle term is the damping force.

In order to get the nonlinear behavior, we should add two physical effects, which are electric and magnetic components of Lorentz force and convective derivative of the electron velocity field [20]: